Thermal conductivity:
-why?
-how?
-what can we get?

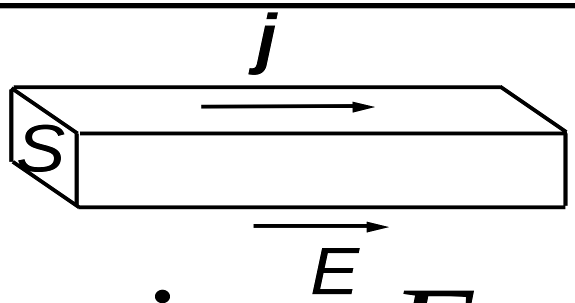
590B

Makariy A. Tanatar

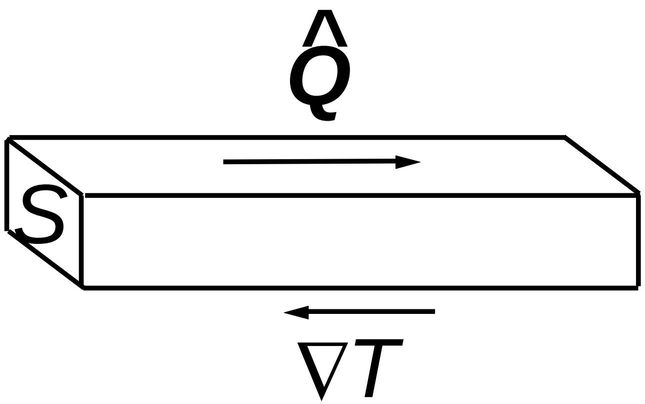
October 2 and 5, 2009

- **Experimental constraints**
- **Experiment design**
- **Dilution refrigerator (DF)**
- **Heat conduction in superconductors**

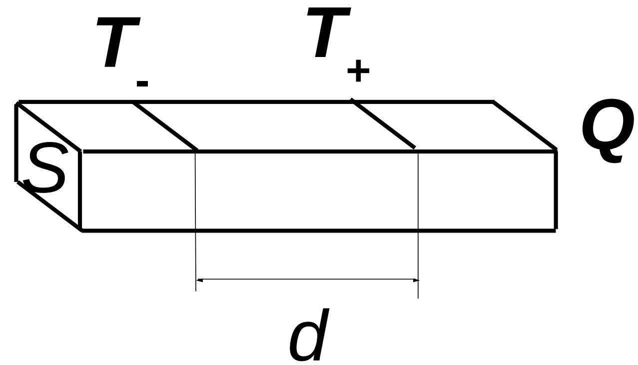
Thermal conductivity: Old problem



$$j = \sigma E$$



$$\hat{Q} = -\kappa \nabla T$$



$$\kappa = \frac{d}{S}\,\frac{Q}{(T_+ - T_-)}$$

Conduction of heat: Old problem

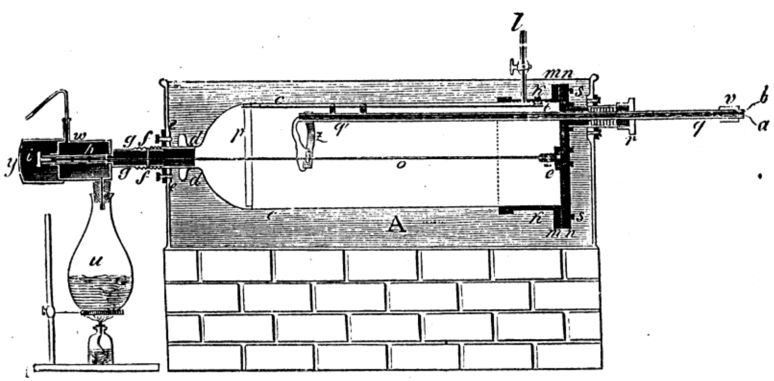
1853. ANNALEN №. 8.
DER PHYSIK UND CHEMIE.
BAND LXXXIX.

I. *Ueber die Wärme-Leitungsfähigkeit der Metalle;*
von G. Wiedemann und R. Franz.

§. 1.

Ueber zwanzig Jahre sind verflossen, seit Hr. Despretz durch seine mühevollen Untersuchungen zuerst einige sichere Zahlenwerthe über die relative Leitungsfähigkeit verschiedener fester Körper für die Wärme aufgefunden hat. —

Die grofse Genauigkeit und Sorgfalt, mit welcher die Versuche von Hrn. Despretz angestellt wurden, hat gewifs mit Recht zur Folge gehabt, dafs die von ihm aufgestellten, nach dem damaligen Zustande der Wissenschaft glänzenden Resultate als Grundlage unserer Kenntnifs in dem bearbeiteten Felde dienen mufsten.



In den Tubulus *d* war ein Messingrohr *ee* eingekittet. In dieses Rohr war bei *ff* ein zweites Rohr *gg* eingeschliffen, welches durch aufgelegte Gummiringe luftdicht daran

150 years of scientific exploration

Wiedemann-Franz law
Good electrical conductors
are good thermal conductors
For metal $\kappa \sim \sigma$

Common experience
Dense crystalline solids
are better heat conductors
than porous materials

Big tables on thermal conductivity
of various materials

Technically important number
Examples

Thermal conductivity: Typical materials

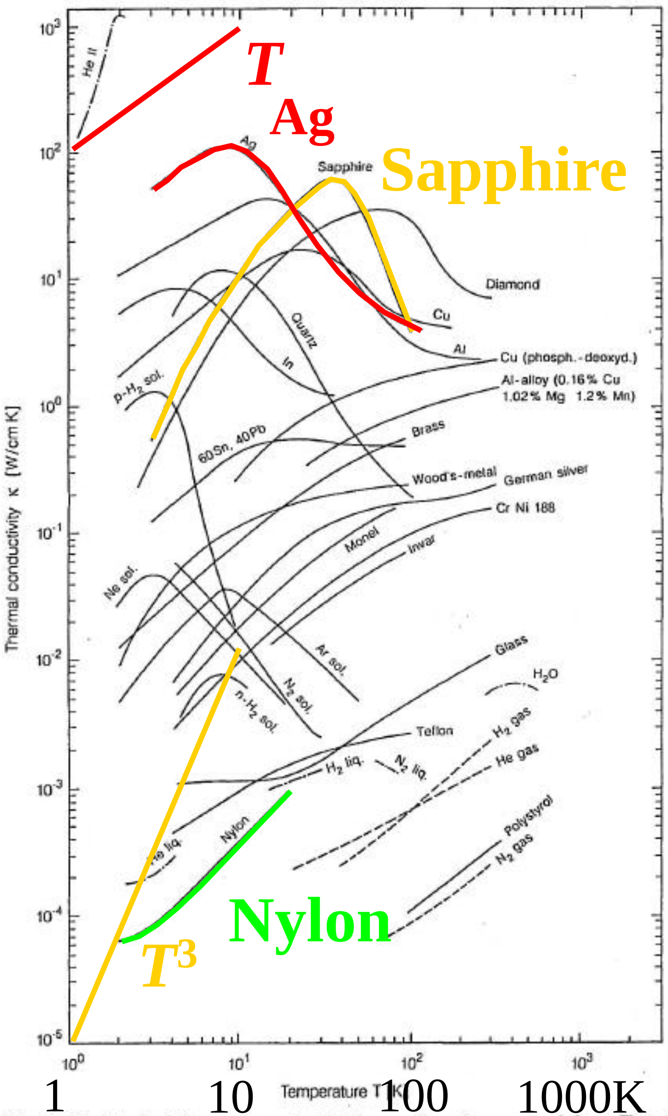
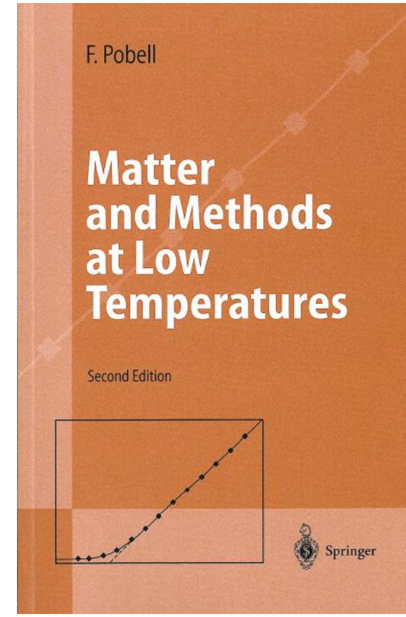


Fig.3.17. Typical thermal conductivities κ of various materials at T > 2 K [3.56-60]. Remember that κ depends on the purity and crystalline perfection of a material

$$\kappa = \frac{1}{3} C v \lambda$$

C- volume specific heat
v- velocity of carrier
sound velocity
Fermi velocity
λ- carrier mean free path



Chapter 3

If λ=const $\boldsymbol{\kappa \sim C}$

Thermal conductivity and heat capacity are closely related

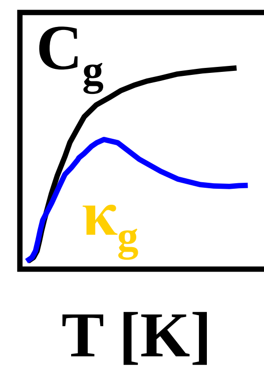
Phonons: for $T<0.1\,\Theta_D$ $C_g \sim T^3$

Electrons: $C_e \sim T$

Low temperatures λ=*const*

Phonons $\kappa_g \sim T^3$

Electrons $\kappa_e \sim T$



Thermal conductivity: Old means well understood!

Lorenz formulation of the Wiedemann-Franz law

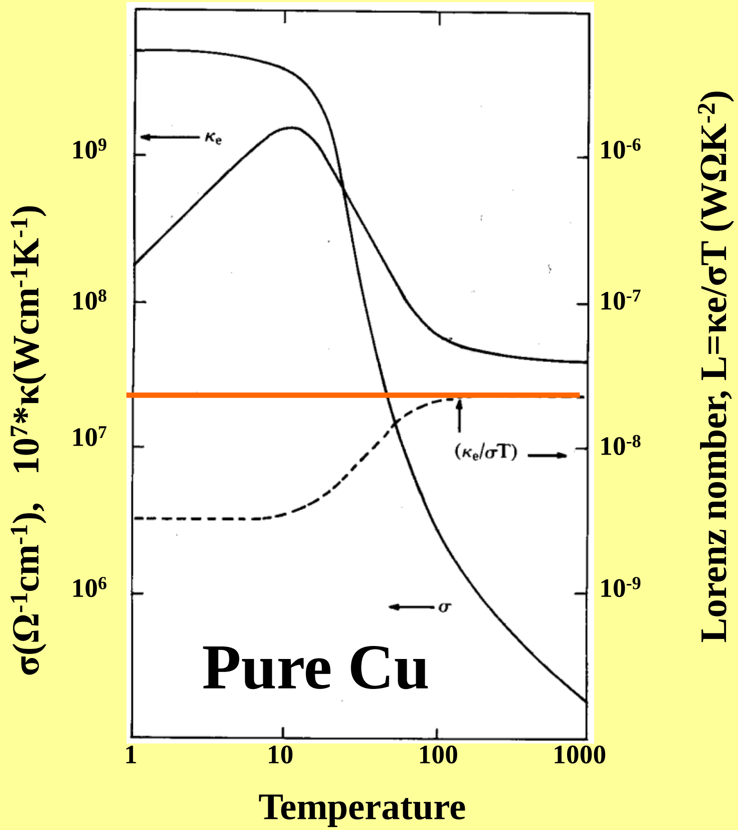
$$L = \frac{\kappa}{T\sigma}$$

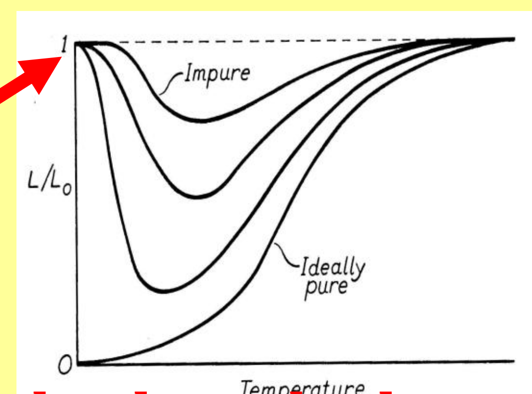
L approximately T-independent Lorenz number

Sommerfeld in 1927 calculated Lorenz number for the particles obeying Fermi-Dirac statistics

$$L_0 = \frac{\pi^2}{3}\left(\frac{k_B}{e}\right)^2$$

Perfect QUANTITATIVE agreement in T→ 0 limit





Why do we bother to study?

Why?!

Quantum states of matter are characterized by different statistics hence different L_0

Examples:

superconductivity

Superconducting condensate carries charge without entropy

superconductors are poor heat conductors $L_0 \equiv 0$

Characteristic behavior for different superconducting pairing states

Quantum Hall effect: charge transport without entropy

new statistics for the quasi-particles

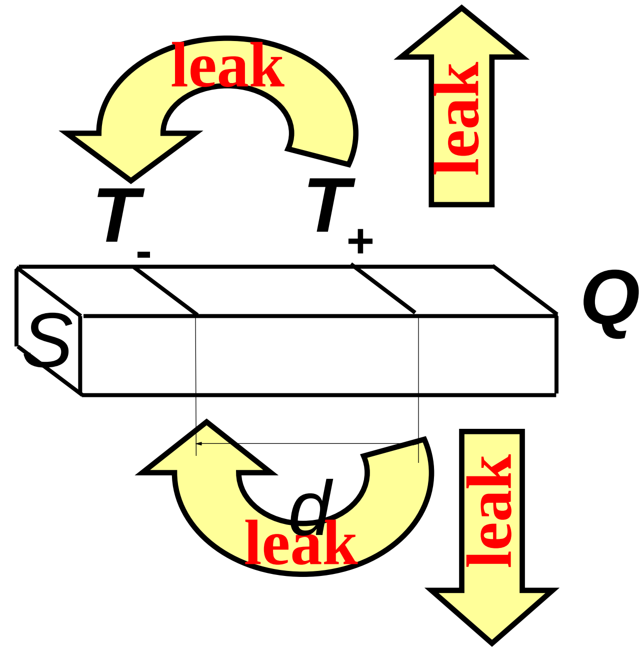
So, because …

Low temperature thermal conductivity is

a semi-quantitative tool for studying quantum states of matter

How? Experimental constraints

Unlike electrical conduction, heat does not need mobile charge carriers and is not confined to electrical wires
Heat flows everywhere!



We want to study.
Heat conduction:
Heat goes through a static material (medium).

We want to exclude:
Heat convection:
Heat goes through a moving medium or is carried away by a moving medium (fluid, gas).
Vacuum ~10^{-6} Torr may be not good enough for poorly conducting samples! M.Y.Choi, P. M. Chaikin, R.L. Greene PRB34, 7727 (86)

Heat radiation:
Heat travels through space with or without a medium.

Thermal radiation: Stefan-Boltzman law

Total energy radiated per unit surface area of a black body in unit time, J^* , is directly proportional to the fourth power of the black body absolute temperature

$$J^* = \sigma T^4$$

$\sigma = 5.67 \cdot 10^{-12}\, W/cm^2 K^{-4}$

Room temperature, $J^* = 5.67 \cdot (300)^4 = 46\ mW/cm^2$

1 K, $J^* = 5.67 \cdot picoW/cm^2$

Is this a lot?

Thermal conductivity: radiation

Room temperature

Substance	κ [mW/cm K]
Silver	4200
Copper	3800
Steel	400
Water	20
Glass	8.4
Wood	1
Wool	0.4
Polyuretane	0.24
Air	0.23

Room temperature,
J^*=46 mW/cm^2

For ΔT=1K and 1 cm^2 area,
ΔJ^* =0.5 mW/cm^2

But this can be not sample but the apparatus surface!

BIG problem for poor conductors!

Degrees of freedom to play
Sample geometry
Quasi-isothermal measurements
Thermal shielding geometry

Thermal conductivity: Dilution fridge

Heat exchange through radiation is not important at 10 K and below.

Cryogenic pumping makes perfect vacuum

Dilution refrigerator sets friendly environment for thermal conductivity measurements*

REVIEW ARTICLE **RSI 51, 1603 (1980)**

Instrumentation at temperatures below 1 K

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Department of Physics and Materials Research Laboratory, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801

(Received 7 July 1980)

This paper, a guide to the literature, is directed to experimentalists planning to extend their research into the temperature range of 0.01–1 K. Included are discussions of refrigeration, thermal contact and isolation, thermometry, and several examples of how standard physical measurement techniques have been adapted to the temperature regime below 1 K.

General problems for all measurements at low temperatures

- RF heating

- Vibration

- Kapitza resistance*

Extremely important for thermal measurements

*Cheese is free only in the mouse trap!

http://www.webelements.com/helium/isotopes.html

Isotope	Atomic mass (m_a/u)	Natural abundance (atom %)	Nuclear spin (I)	Magnetic moment (μ/μ_N)
^{3}He	3.016 029 309 7(9)	0.000137 (3)	$^{1}/_{2}$	-2.127624
^{4}He	4.002 603 2497(10)	99.999863 (3)	0	0

Physical properties

	Helium-3	Helium-4
Boiling (1atm)	3.19 K	4.23 K
Critical point	3.35 K	5.19 K.
Density of liquid (at boiling point, 1atm)	0.059 g/ml	0.12473 g/ml
Latent heat of vaporization	0.026 kJ/mol	0.0829 kJ/mol

Cooling power of evaporating cryogenic liquid

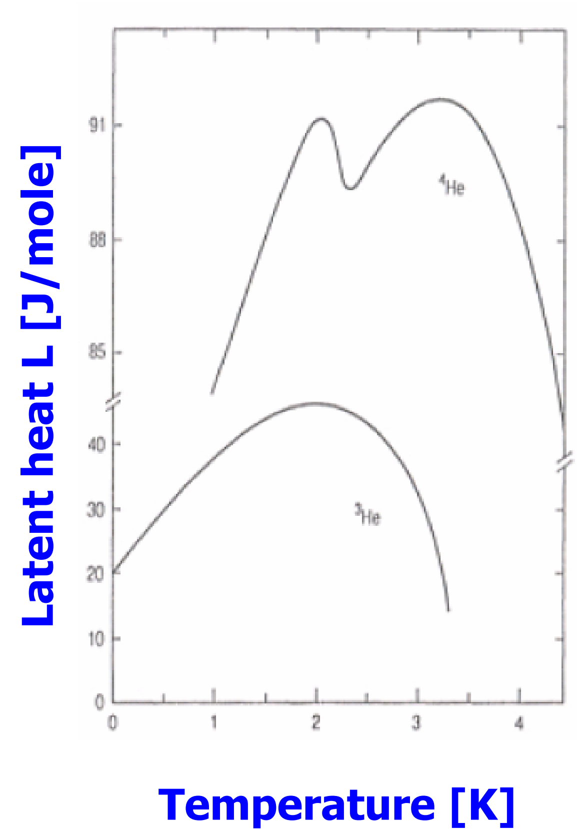
$Q = n\Delta H = nL$,

L approximately constant

Q cooling power
n rate of evaporation, molecules/time
ΔH enthalpy of evaporation
L latent heat of evaporation

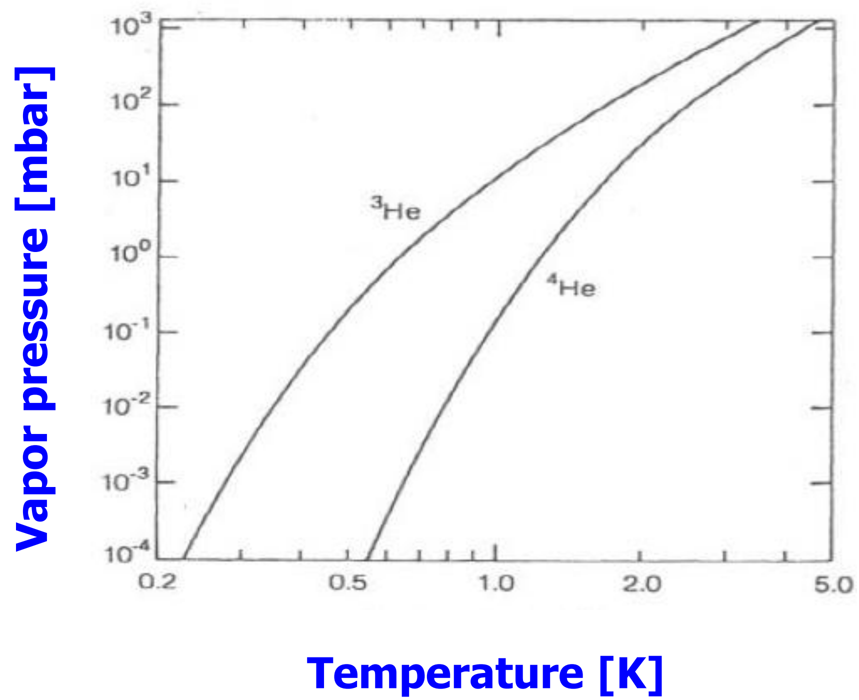
For a pump with constant volume rate V

$Q = VP(T)L$



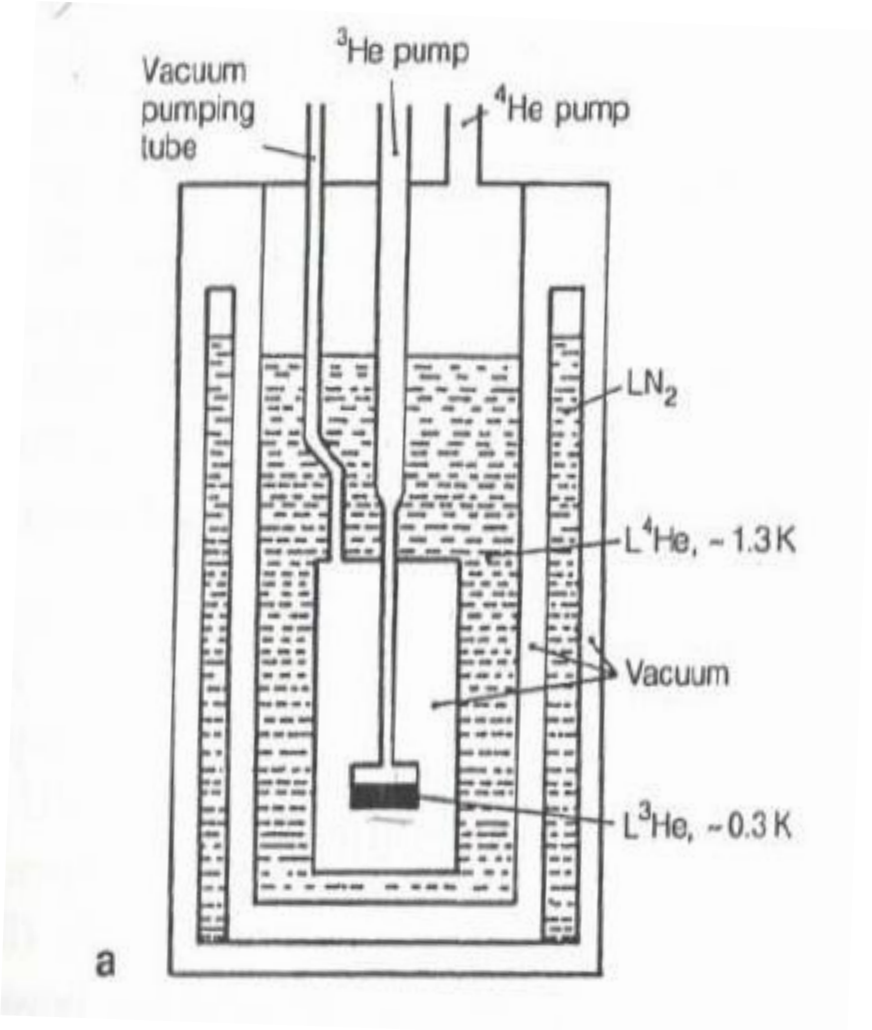
Cooling power proportional to vapor pressure
Q~P(T)~exp(-1/T)

Exponentially small at low T
We can get by pumping on
He-4 T~1K
He-3 T~0.26 K



Evaporative cooling is used in
1K pot
He-3 cryostat

He-3 refrigerator

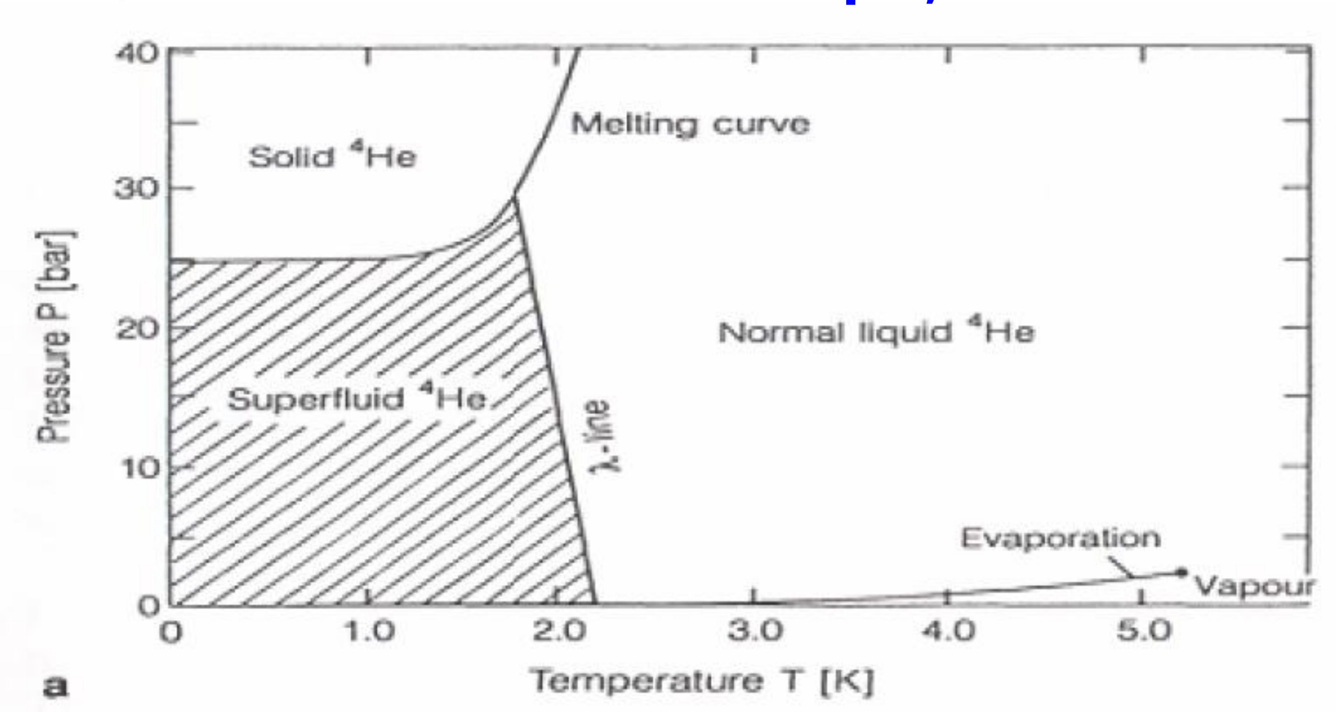


Typical features:
Sample in vacuum
(rarely sample in He-3 liquid)
One shot mode of operation
Hold time 10-60 hours

Operation sequence:
Release He-3 from cryopump
Condense by heat exchange with 1K pot
Cool condensate to 1.5K
Start cryopumping to reach base temperature

He-3 is stored in a sealed space to avoid loss
He-3 pump is called Sorb, uses cryopumping

He-4 nucleus has no spin, Boson

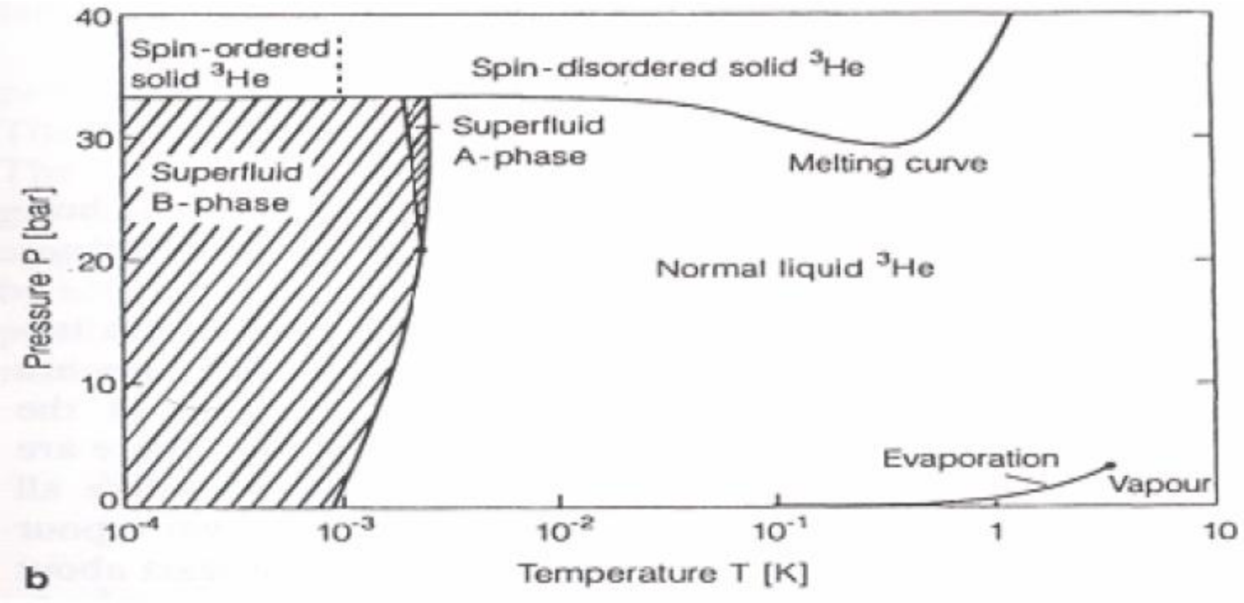


No solid phase due to:
weak van der Waals inter-atomic interactions, E_{pot} is low
Large quantum mechanical zero-point energy $E_0=h^2/8ma^2$
due to small mass, E_{kin} is high
Bose-Einstein condensate instead of a solid

Quantum liquids, ratio $\lambda=E_{kin}/E_{pot}$ He4 $\lambda=2.64$ He3 $\lambda=3.05$
Amplitude of vibrations about 1/3 of interatomic space

He-3 nucleus has spin 1/2, Fermion

Additional spin entropy



Bose-Einstein condensate of pairs, several superfluid phases

Special feature: Below T_F spins in the liquid phase are spatially indistinguishable.
Therefore they start obeying Fermi statistics and are more ordered than in the paramagnetic solid phase!

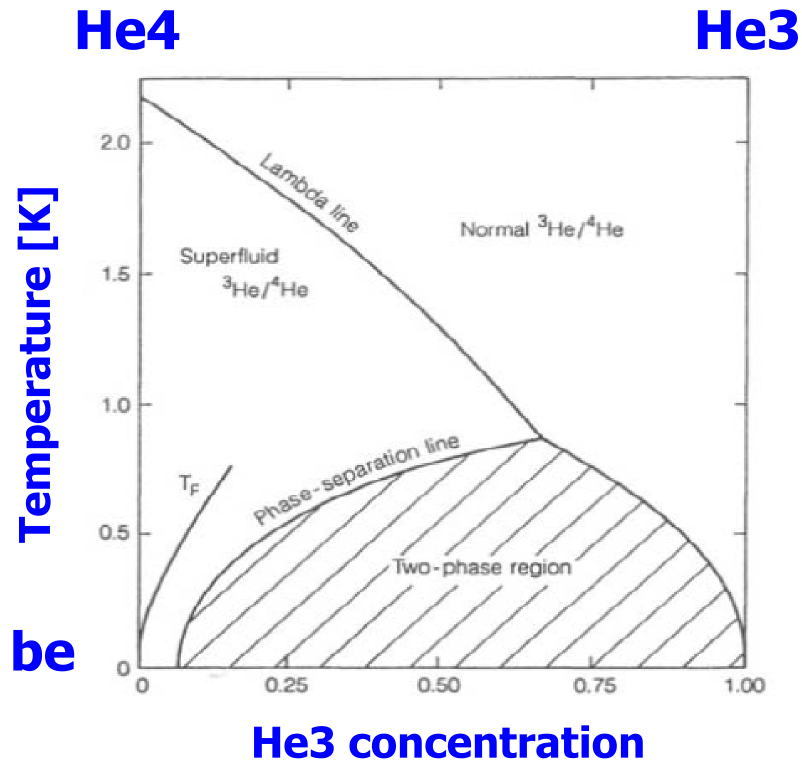
Mixture of He3 and He-4

Phase separation of the mixture into He3 rich and He3 poor phases, but not pure He3 and He4

Pure quantum effect
classical liquids should separate into pure components to obey 3rd law of thermodynamics, S=0

In case of He3-He4 mixture, S=0 can be for finite concentration because of the Fermi statistics for He3 and Bose statistics for He4

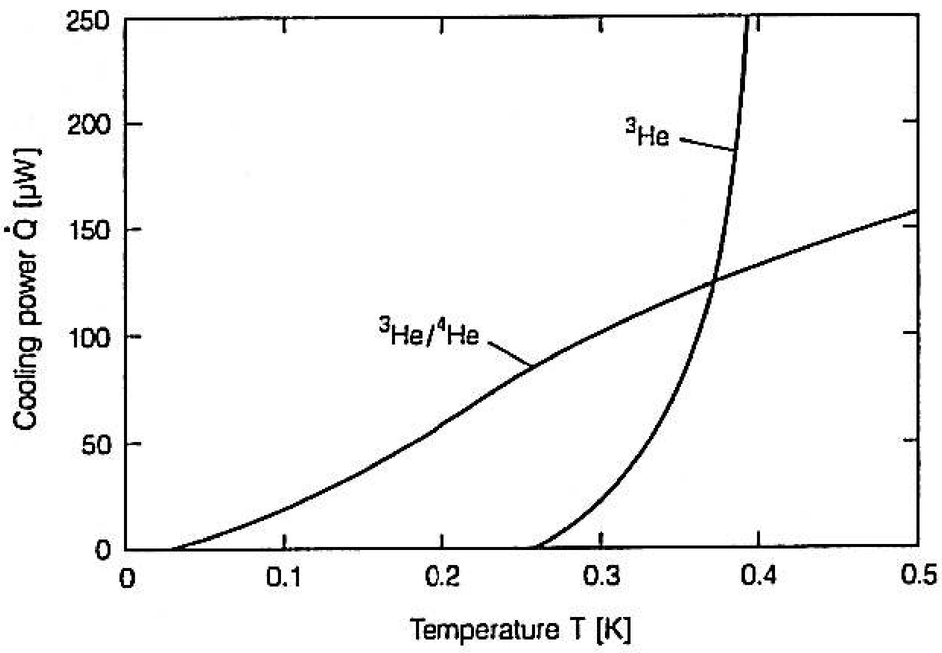
Phase separation starts below T=0.867 K (max at X=0.675)



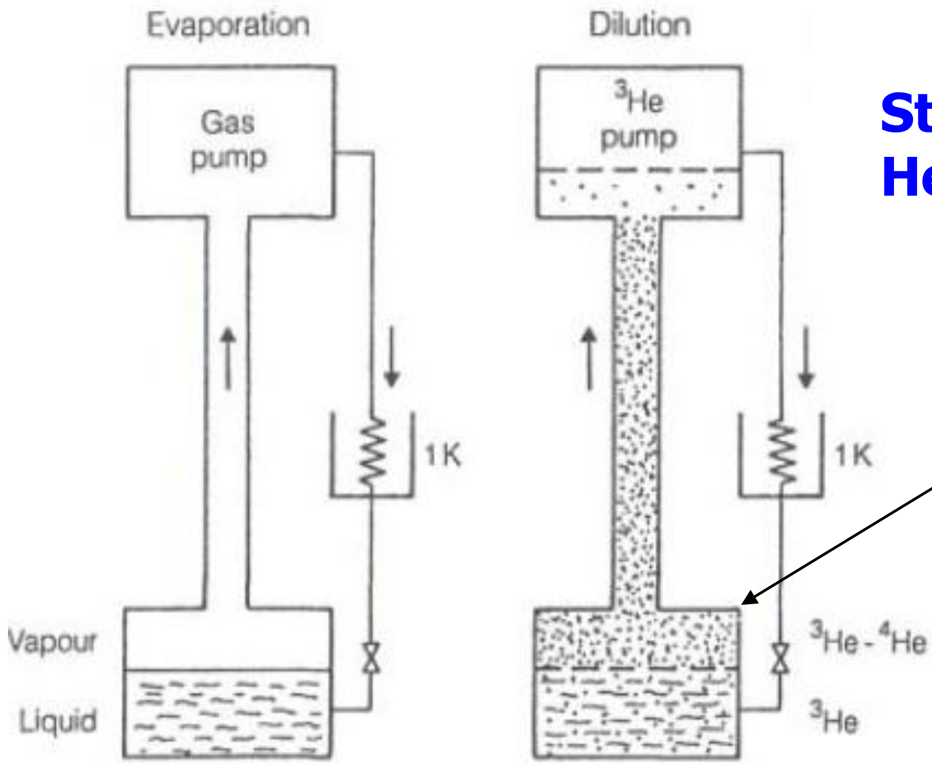
Cooling power:
Power law decrease
Instead of exponential decrease

Enthalpy of mixing uses the difference
In specific heat of two phases

$\Delta H = \int \Delta C \, dT$
$Q \sim x \, \Delta H \sim T^2$



Evaporative cooling **Dilution**

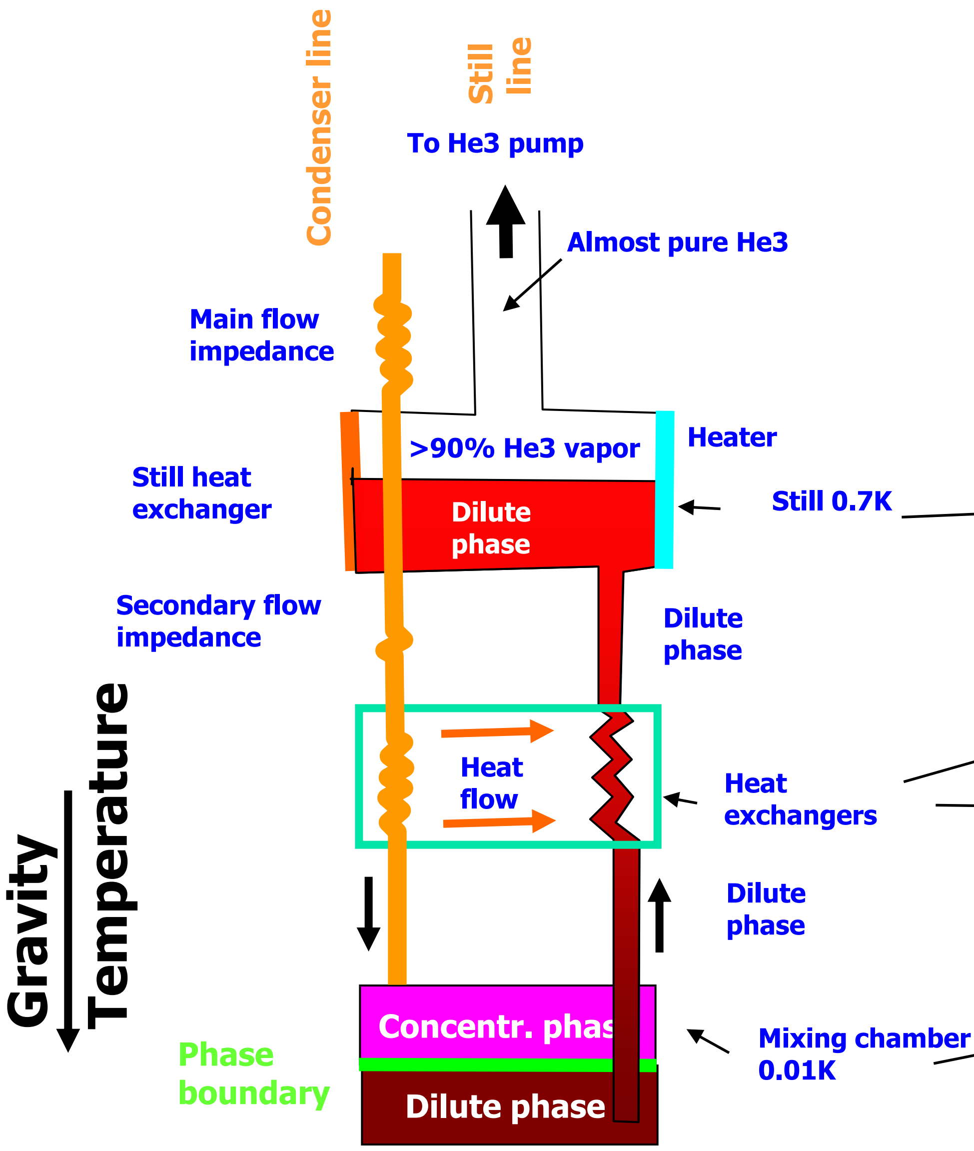


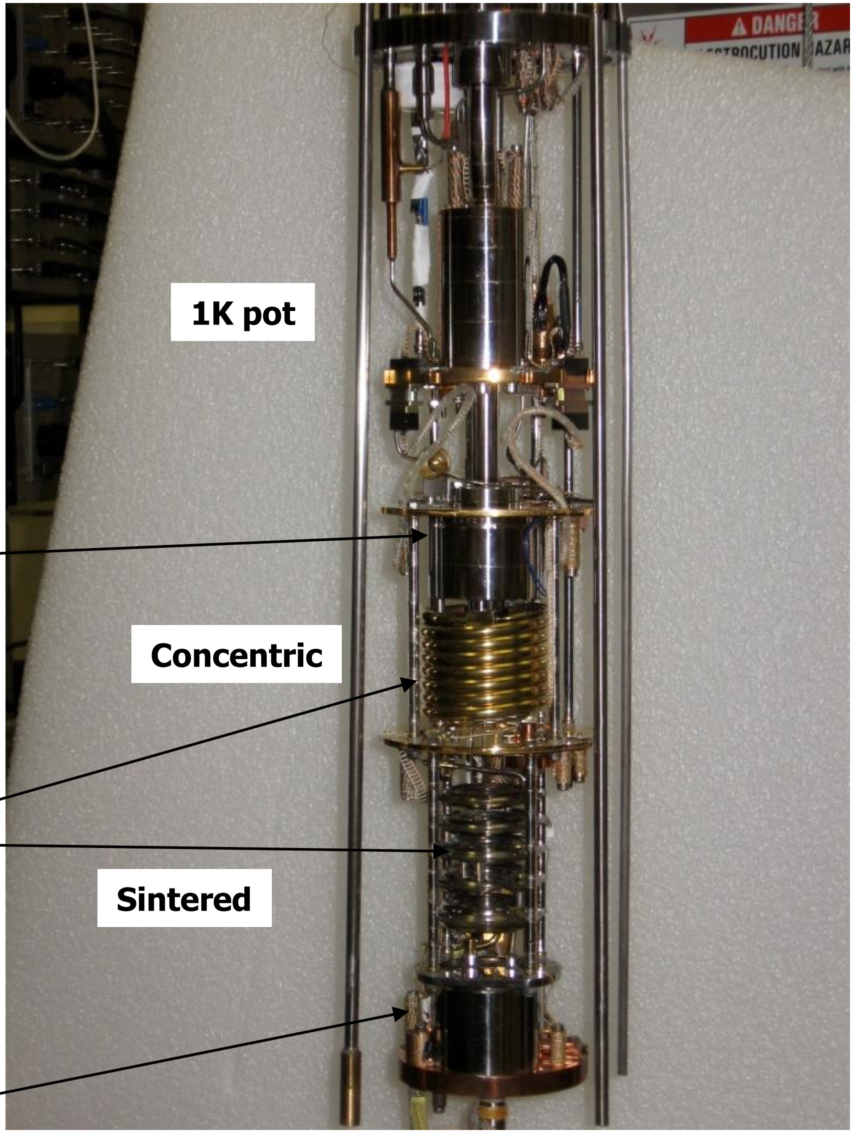
Still evaporates He3 from mixture

Mixing chamber

Phase separation line

Dilution Refrigerator: more details

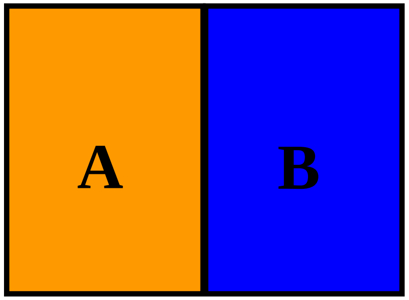




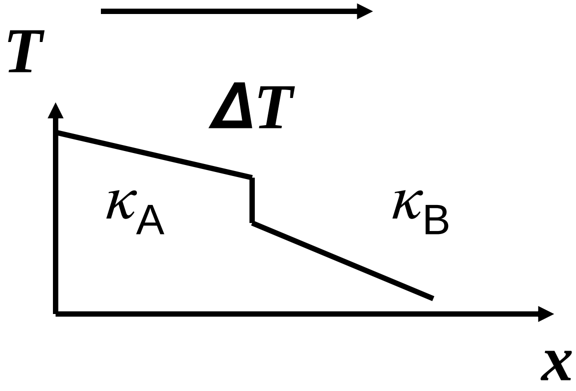
Kapitza resistance

A discontinuity in temperature across the interface of two materials through which heat current is flowing

acoustic impedance mismatch at a boundary of two substances
phonons have probability to be reflected



$$\hat{Q} = \text{heat current density}$$



Kapitza resistance, **$\sim T^3$**

Important effect as *T* tends to 0

1K vs 10 mK

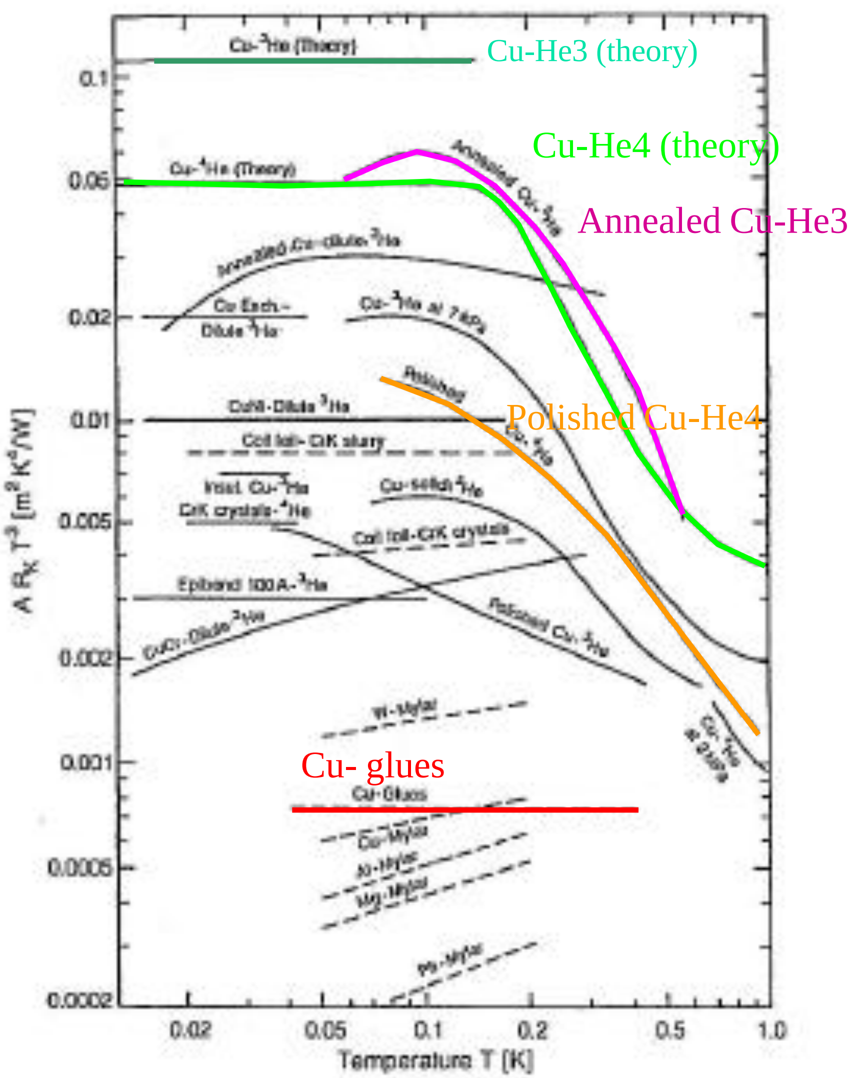
6 orders of magnitude change!

$$\hat{Q} = \kappa_K \Delta T$$

κ_K- Kapitza conductance

Good Thermal contact at low temperatures needs conduction electrons

Kapitza resistance



Mismatch of the excitation spectra

R_K dissimilar> R_K similar

Pobell's recommendation

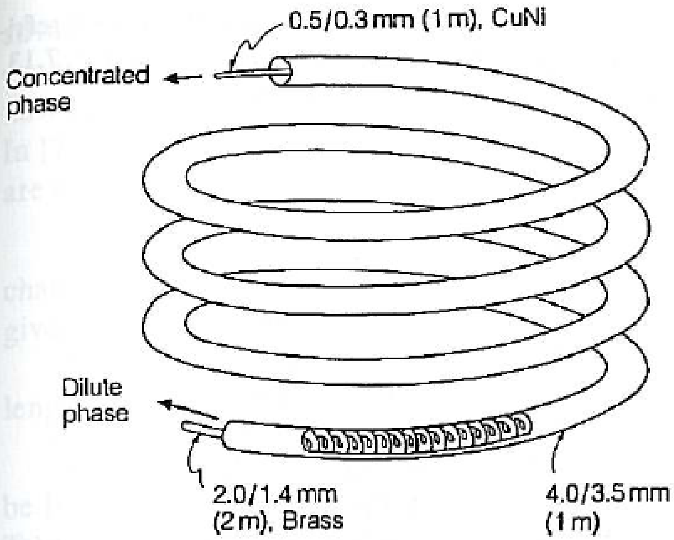
Metal-metal
Polished surfaces
Strong mechanical force
Gold-plating
Welding

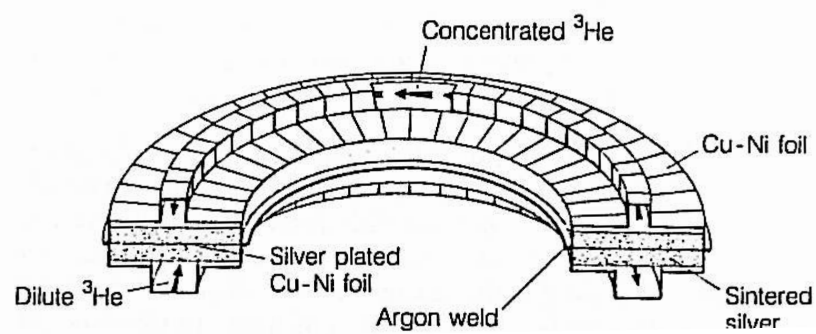
Soldering
Silver based allows
Few solders are not SC!

Insulators
GE
Stycast 1266

Dilution refrigerator: heat exchanging

Need big surface area contacts!





Concentric heat exchanger
High temperatures

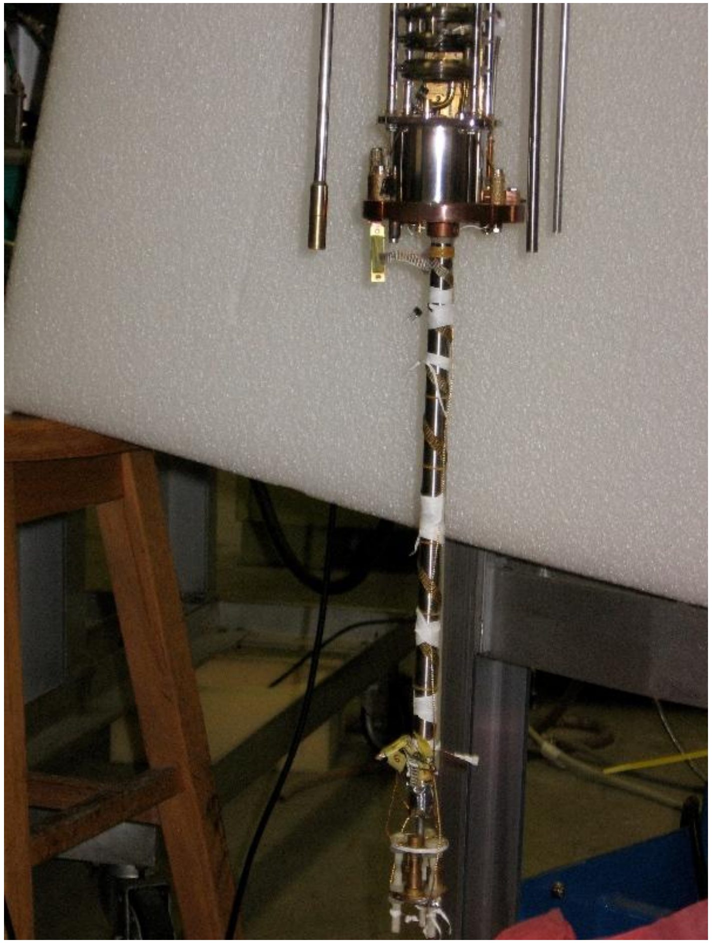
Welded Cu-Ni foil
Sintered submicron silver powder
Close to mixing chamber

Dilution refrigerator: Experiment cooling

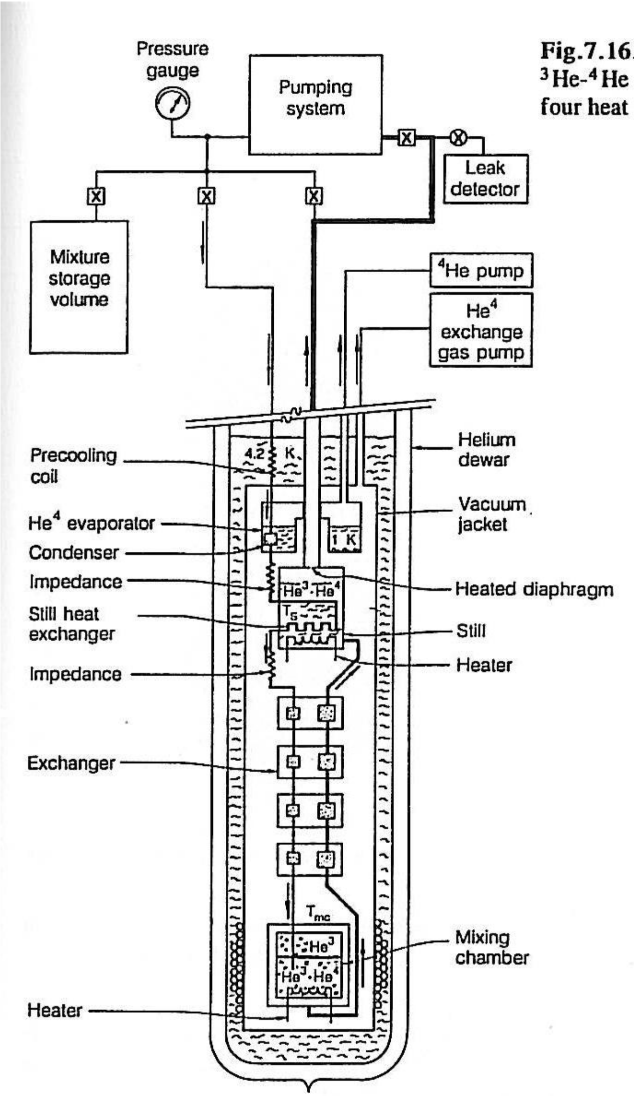
Do not rely on insulating contacts!

Vibration

RF heating



Dilution refrigerator: gas handling at room temperature

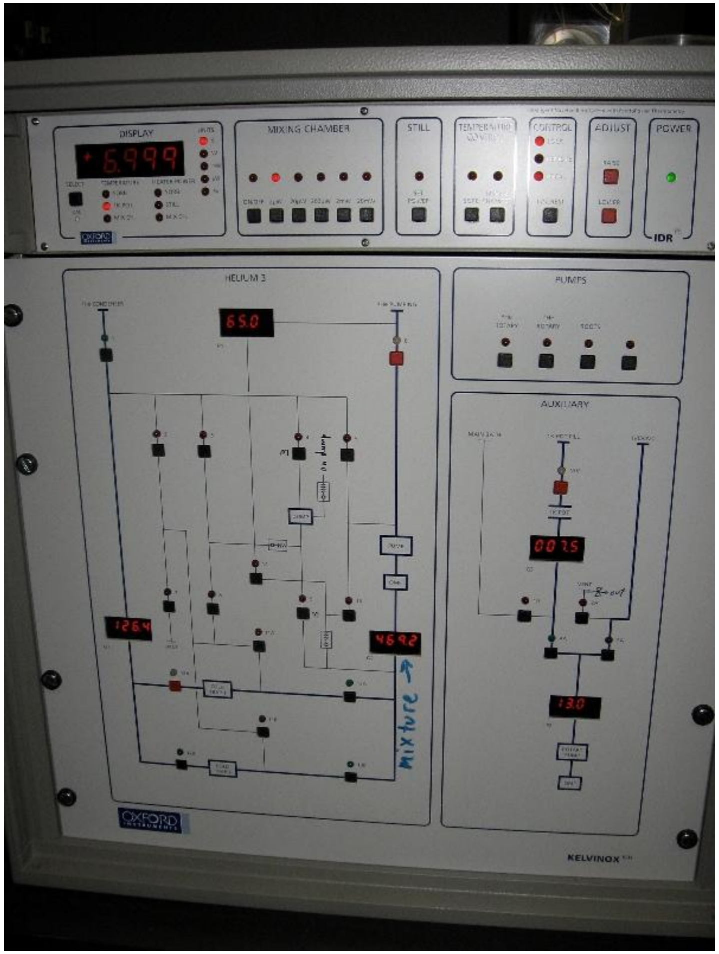


Key elements:
He3-He4 Gas storage "Dump"
Vacuum pump for 1K pot
Vacuum pump for He3 circulation
Roots (booster) pump for Still line pumping
Cold traps for mixture cleaning

Very demanding to vacuum leaks
To avoid loss of mixture, all operation goes at $P<P_{atm}$
Leaks in, not out!

Dilution refrigerator: gas handling system

Front view **Back view**



He3 **He4**

Kapitza resistance in the bulk: electron-phonon decoupling

Phonons vs electrons
Bosons vs Fermions

Contact **Sample**

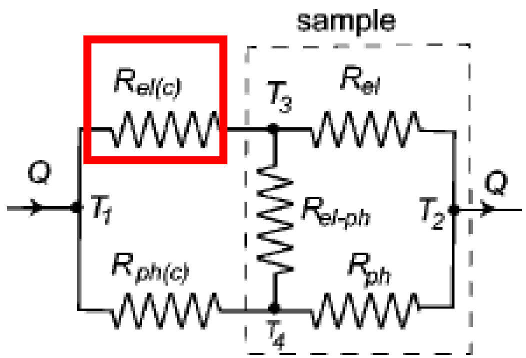
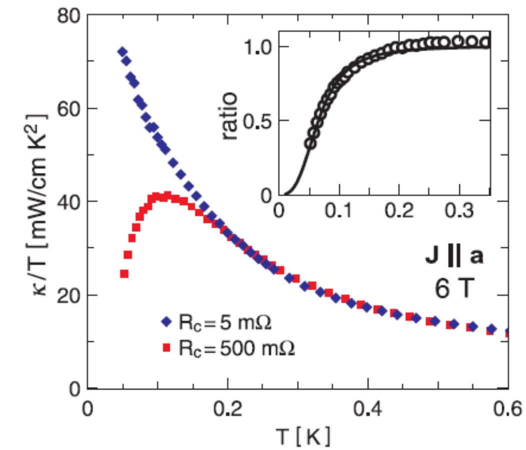


FIG. 1. A simple picture of the experimental configuration for thermal conductivity measurements. The thermal resistance to phonon and electron heat currents that occurs before the current enters the sample are represented by $R_{ph(c)}$ and $R_{el(c)}$, respectively. The thermal resistance to phonon and electron heat flow through the sample are represented by R_{ph} and R_{el}, respectively. The electron-phonon heat transfer rate is associated with $R_{el\text{-}ph}$. Temperatures at different positions have been labeled.

$$\mathbf{R_{el\text{-}ph}^{-1} \sim K_{el\text{-}ph}} \boldsymbol{T^n}, \boldsymbol{n}=4\text{-}5.$$



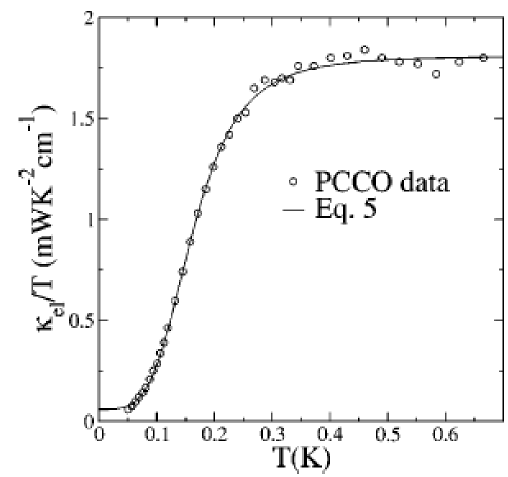
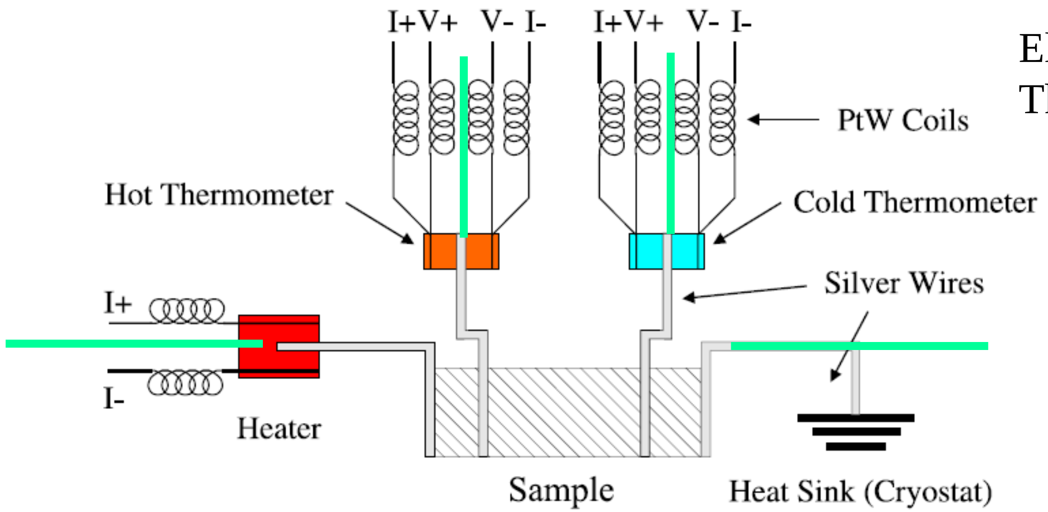


FIG. 2. The electronic thermal conductivity κ_{el}/T measured (Ref. 1) for normal state $Pr_{2-x}Ce_xCuO_{7-\delta}$ is plotted versus T along with the result of Eq. (5).

M.Smith et al. PRB71, 014506 (2005)

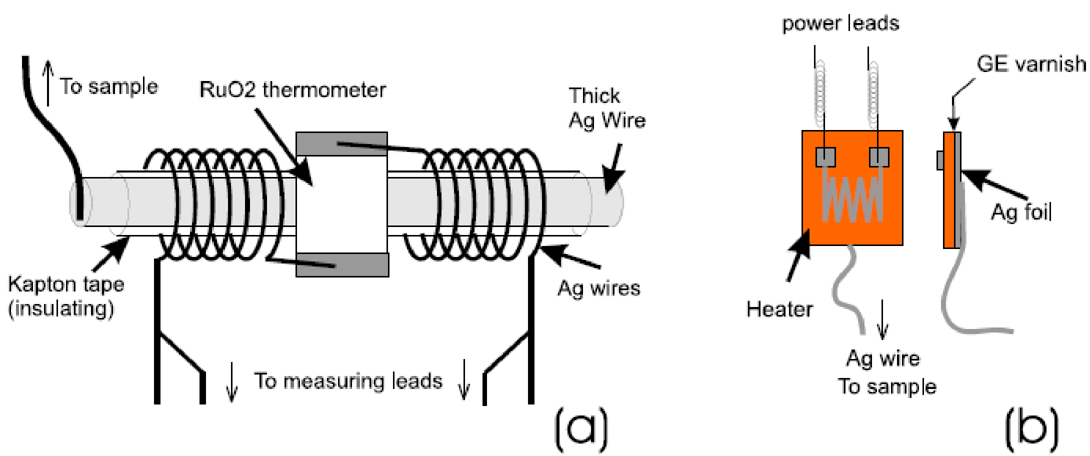
How? Thermalization

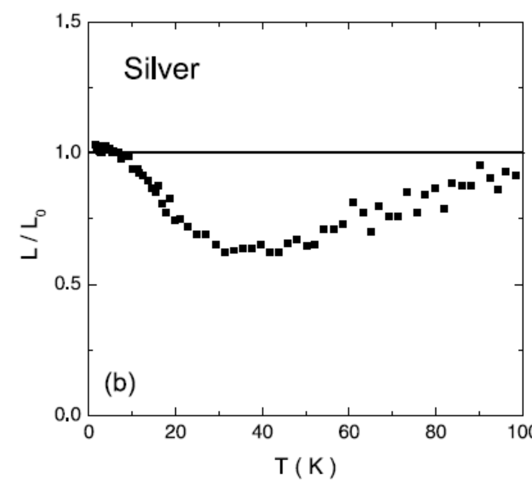


Electrical connection
Thermal insulation

4 probe
electrical resistivity
thermal conductivity
Thermopower
But never the same
Sample wires!

Test results





How? Sample preparation and mounting

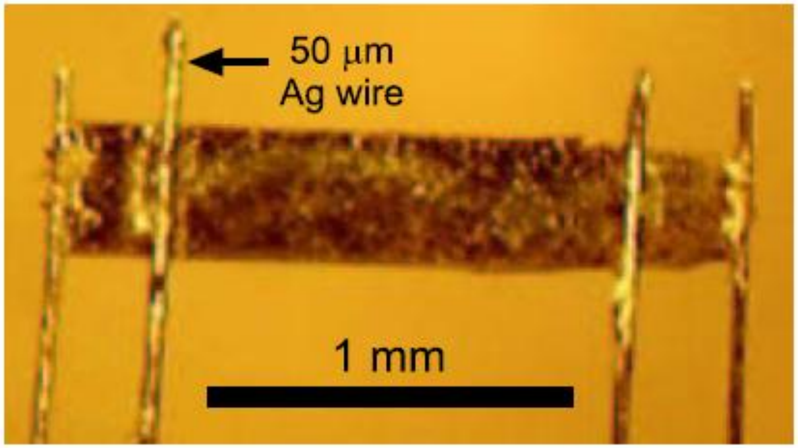
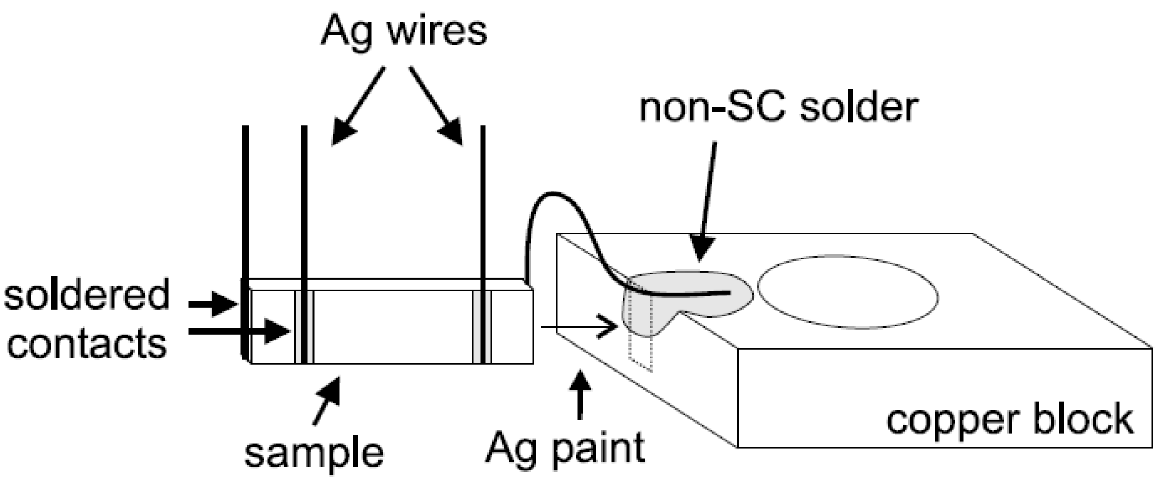


Figure 4.1: $CeCoIn_5$ sample with In-soldered leads.



Something about superconductivity

BCS theory of superconductivity

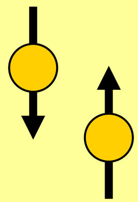
conventional
unconventional

superconductivity

Thermal conductivity
of superconductors

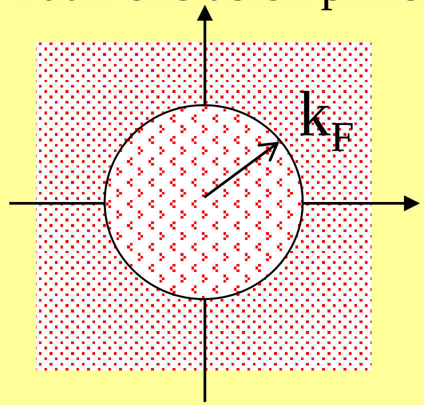
T zero field
Jφθ anisotropy
X impurities
H vortex state
Hφθ anisotropy
Hc2

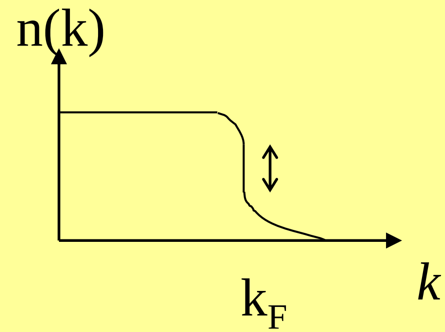
Metallic state



fermionic quasiparticles

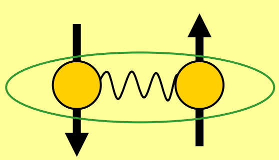
Pauli exclusion principle





finite attraction

Superconducting state

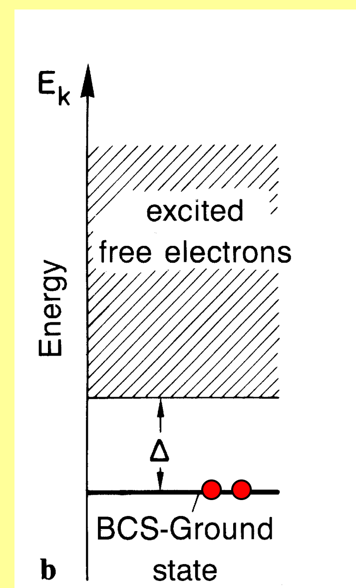


composite boson
"Cooper pair"
$\boldsymbol{k}_1 = -\boldsymbol{k}_2$

Cooper pair – boson
All occupy same
quantum state

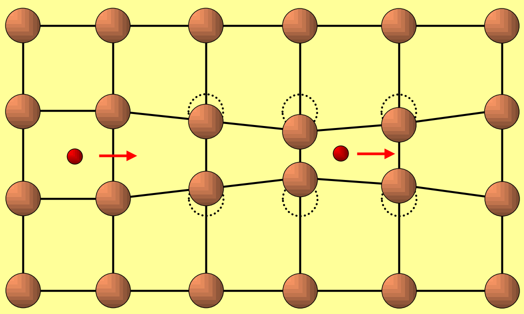
BCS coherent groundstate
- pairwise correlated
NO one-electron picture

$$\Psi(r) \sim \Delta(r)$$



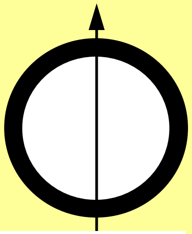
energy
break
pair
2Δ

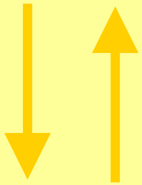
BCS model – 'conventional' superconductivity



phonon mediated coupling

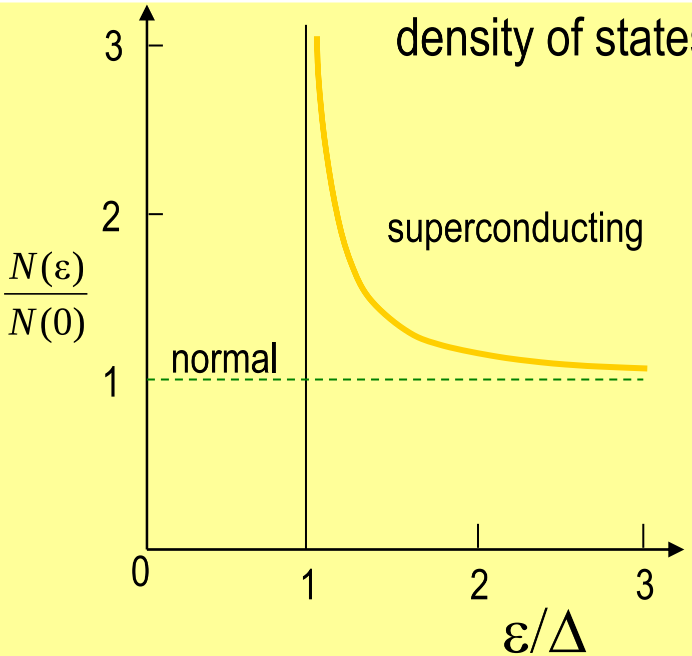
energy gap Δ, Ψ(r)





spin singlet

Experimental Manifestation – fully gapped superconductor



For low T expect

Activated behaviour

e.g. $C_e \sim \exp\left(-\frac{\Delta}{k_B T}\right)$

Minimal effect of impurities unless magnetic

Heat conduction conventional SC: electrons

Electronic excitations freeze out,
Condensate carries no entropy κ≡0

SC=thermal insulator

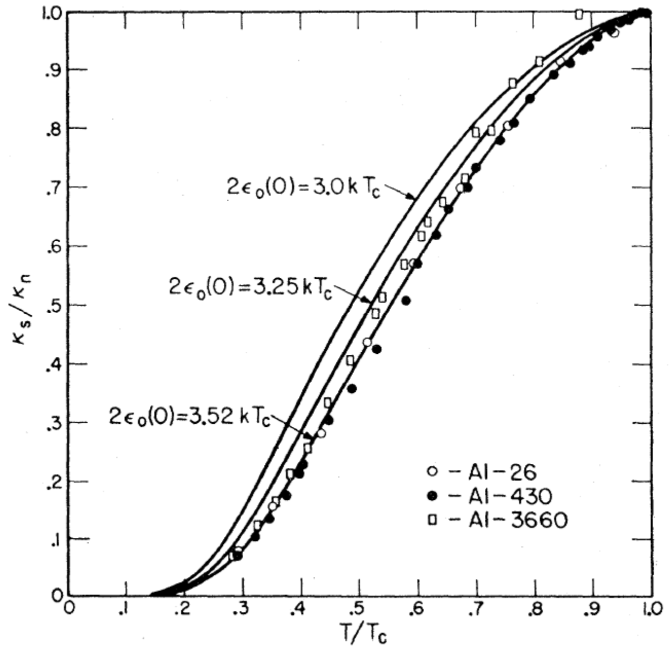


Fig. 3. Ratio of superconducting to normal thermal conductivity for aluminum.

C. B. Satterthwaite, Phys. Rev. **125** 873 (1962)

BRT theory

J. Bardeen, G. Rickayzen, L. Tewordt (1959)

Low T, ~ρ_0 regime (σ$_e$~const)

$$\kappa_e = (nt_{tr}T/2m)\int_{\Delta/T}^{\infty} x^2 ch^{-2}(x/2)dx$$

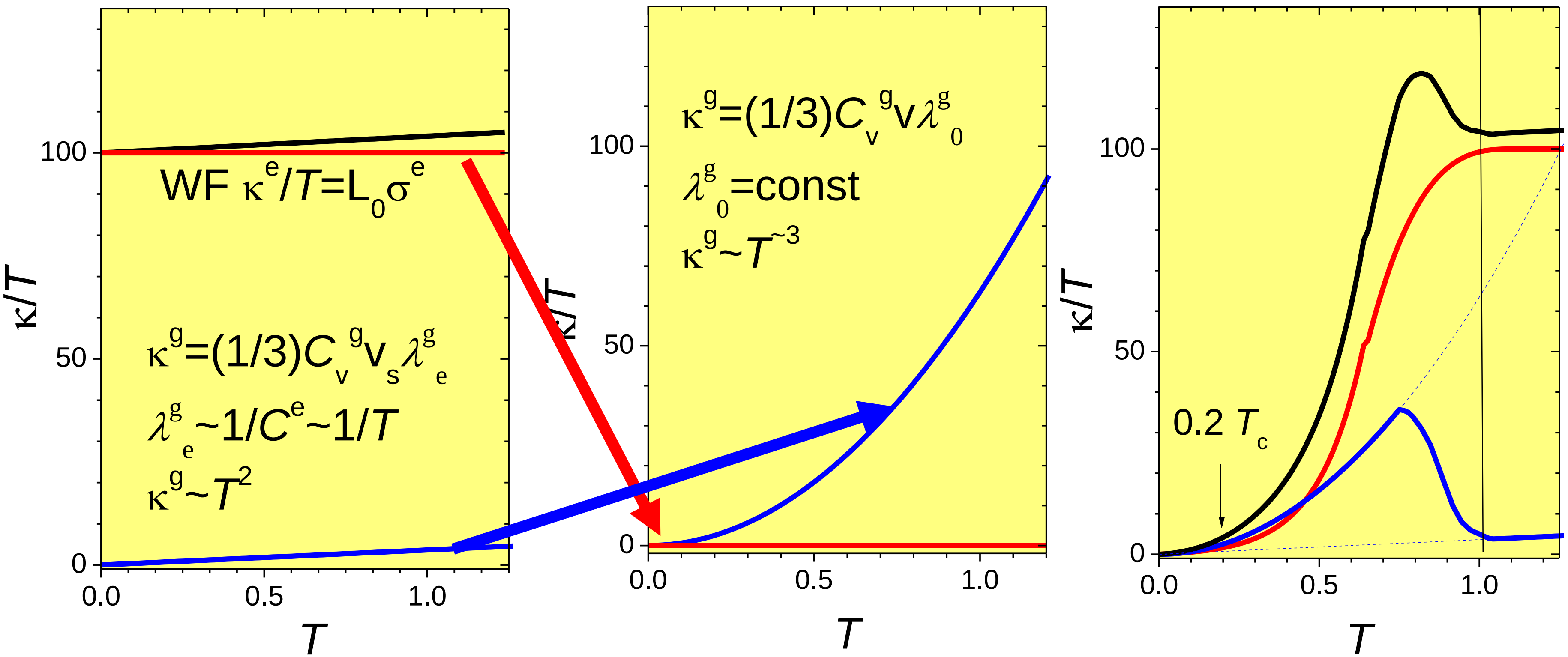
$$T \ll T_c$$

$$\frac{\kappa_{es}}{\kappa_n} \propto \left(\frac{\Delta}{k_B T}\right)^2 \exp\left(-\frac{\Delta}{k_B T}\right)$$

+phonons!

The physics of heat conduction: metals, insulators, SC

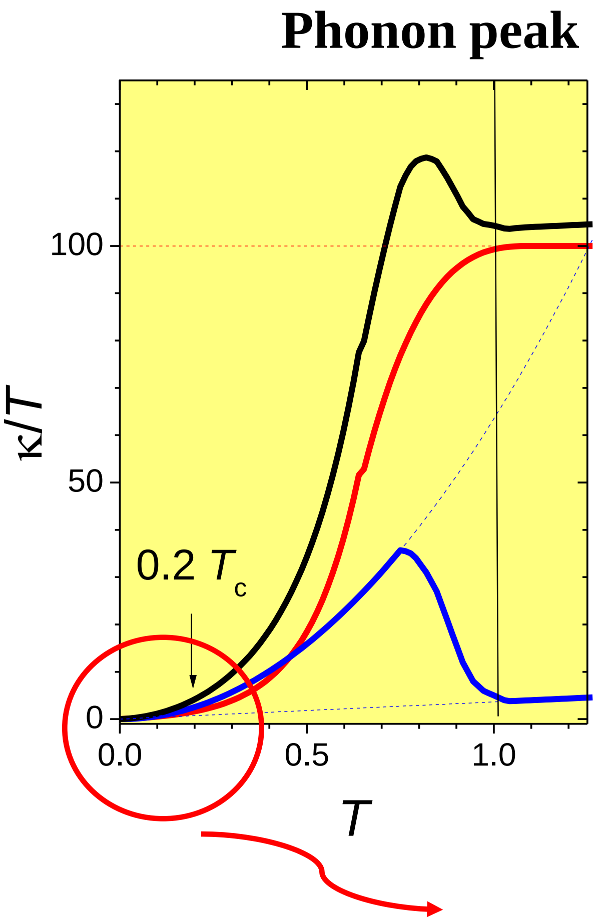
Metal **Insulator** **SC**



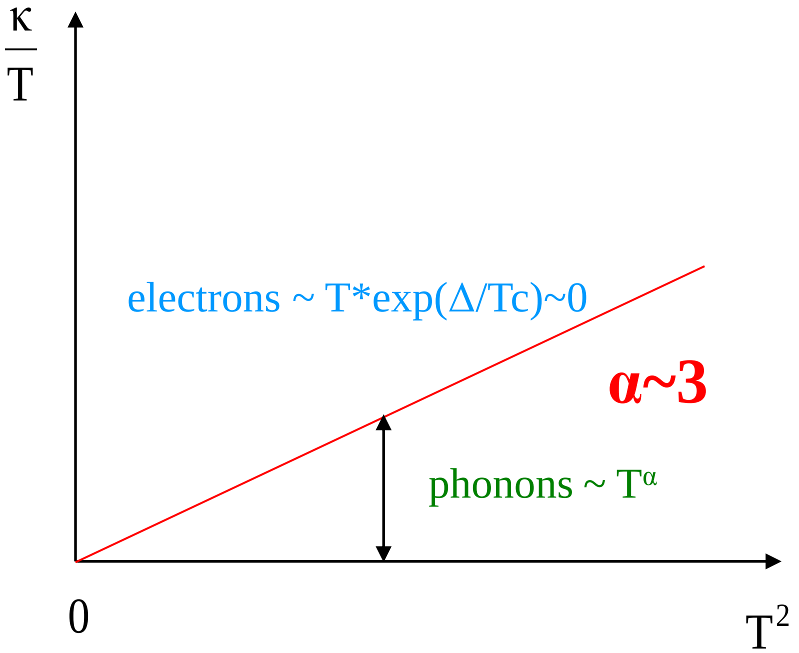
Phonon scattering on conduction electrons dies

In experiment we study thermal metal-insulator crossover

The physics of heat conduction: superconductors T <<Tc



$$\frac{\kappa_{es}}{\kappa_n} \propto \left(\frac{\Delta}{k_B T}\right)^2 \exp\left(-\frac{\Delta}{k_B T}\right)$$



Heat conduction conventional SC: magnetic impurities

With pairbreaking
Gap in DOS is filled

Gapless: TC metal with reduced DOS

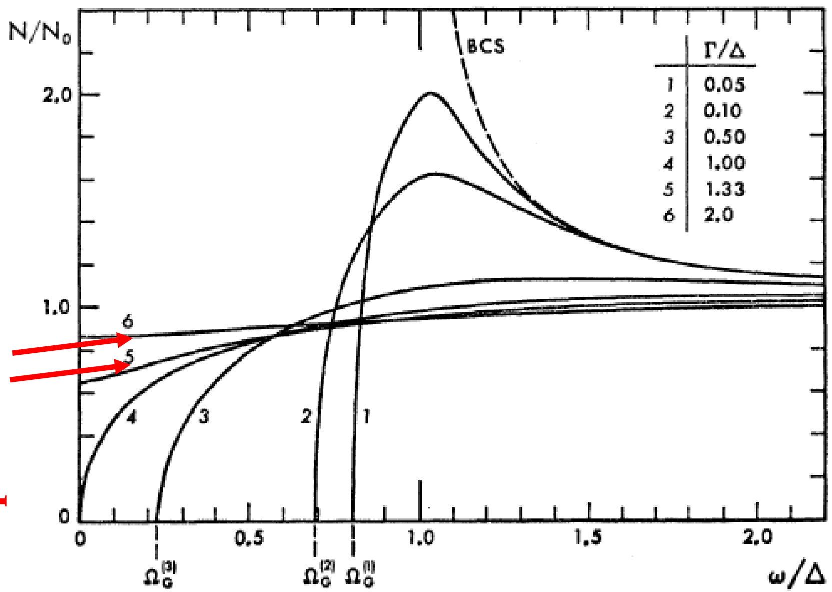


FIG. 6. The normalized density of states N/N_0 plotted as a function of the reduced quasiparticle energy for several values of the reduced inverse collision time Γ/Δ (from Skalski *et al.*, Ref. 7). The corresponding values of the gap Ω_G are indicated.

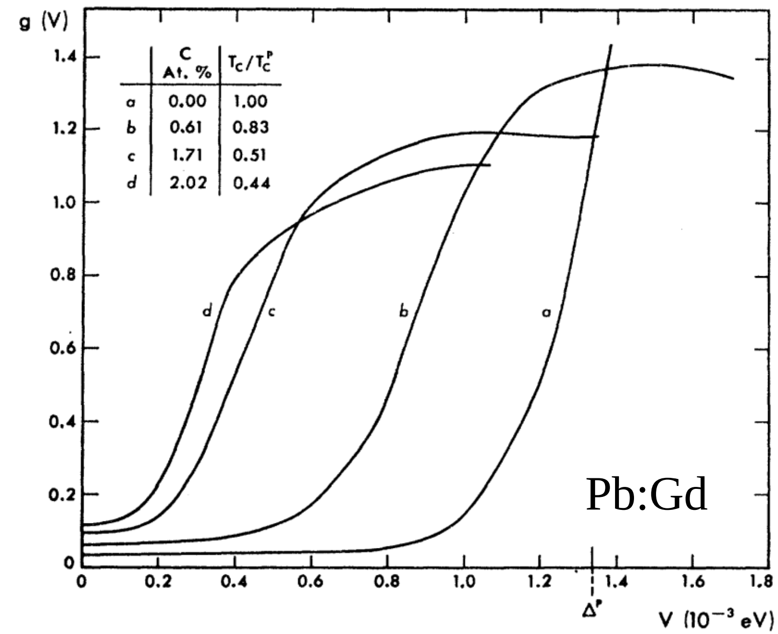


FIG. 2. Measured tunnel conductance $g(V)$ of the Pb-Gd system as a function of energy V for various Gd concentrations c. Curves (a) and (b) were taken at 1.1°K; curves (c) and (d) at 0.4°K. The transition temperature of the pure superconductor is denoted by T_c^P. [The finite values of $g(V)$ near $V=0$ are unreliable; see text.]

Wolf PR137, 557 (1965)

Magnetic pair-breaking smears the SC gap edges
Finite density of states in the SC gap

Heat conduction conventional SC: magnetic impurities "Gado"

Th:Gd

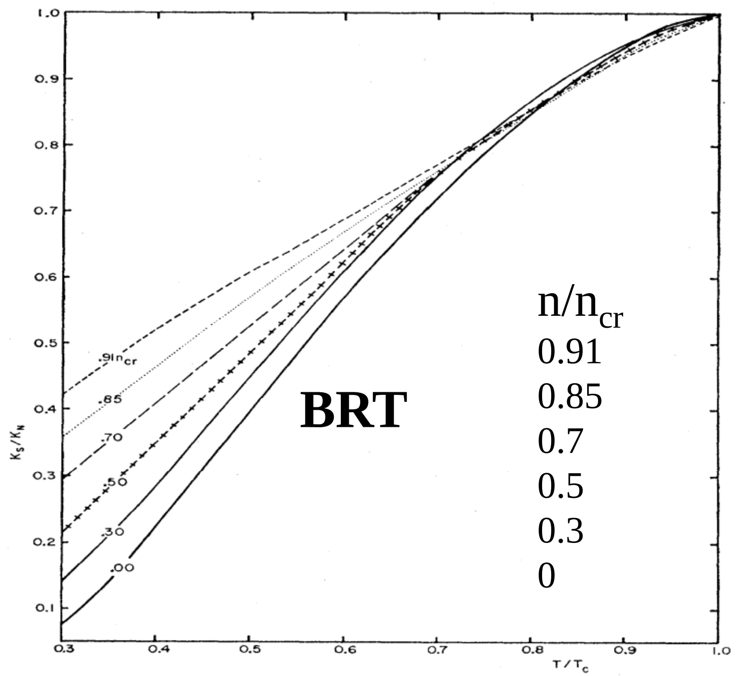


Fig. 3. The ratio of the electronic thermal conductivity in the superconducting and normal states (K_s/K_n) vs $t \equiv T/T_c$ for various paramagnetic impurity concentrations. T_c is the transition temperature for the relevant impurity concentration, the latter being expressed in terms of n_{cr}, the concentration required to completely destroy superconductivity. The curve for nonmagnetic impurities alone is denoted by $n_i = 0.00 n_{cr}$. The $n_i = 0.70 n_{cr}$ curve is almost identical to the 0.85 curve for $t \gtrsim 0.7$ and is not shown explicitly in this region. These results are based on (1.1) or (3.11).

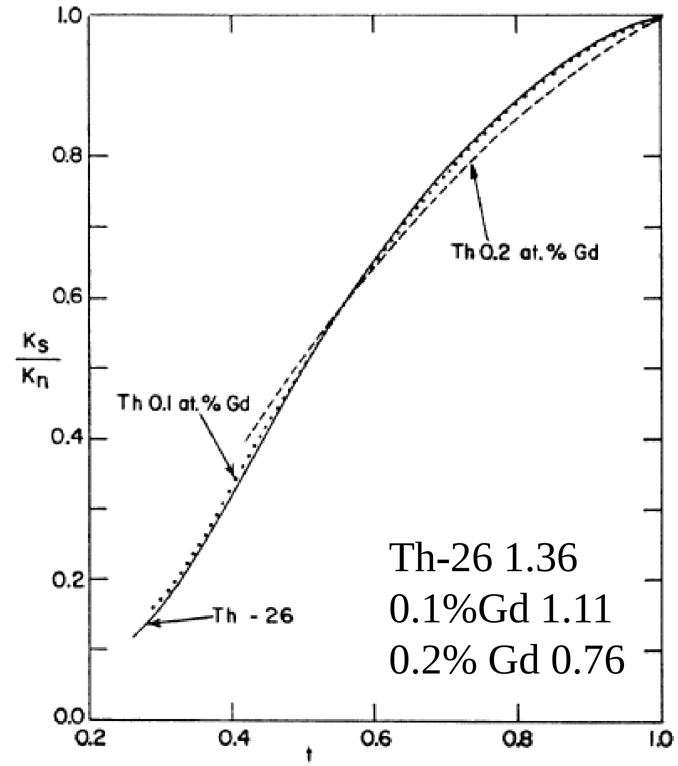


Fig. 6. Comparison of pure Th with the Th-Gd alloys. At high temperatures the alloy data lie below pure Th, and at low temperatures they lie above the pure-Th data.

V. Ambegaokar, A. Griffin
PR137, 1151 (1965)

R. L. Cappelletti, D. K. Finnemore
PR188, 123 (1969)

Localized states?

Heat conduction conventional SC: magnetic impurities

Pb:Gd

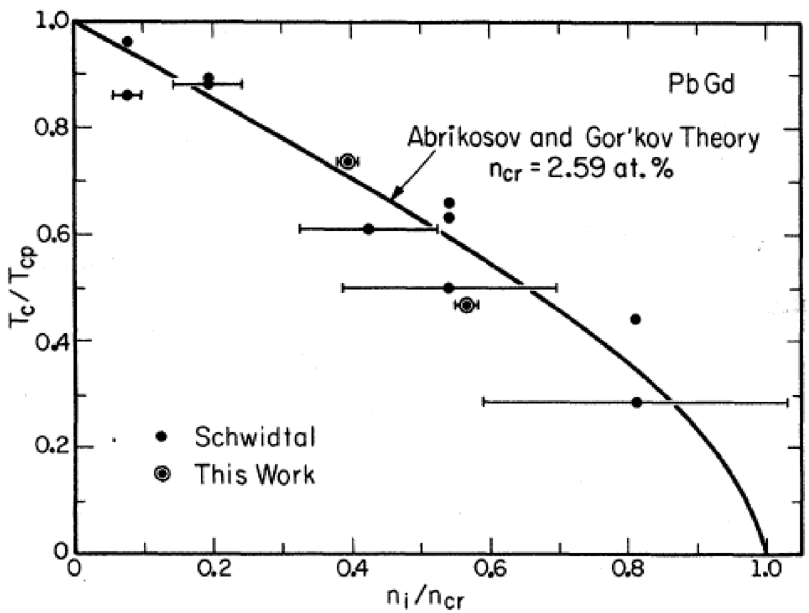


FIG. 4. Experimental and theoretical (Ref. 2) reduced transition temperature as a function of the reduced impurity concentration for the Pb-Gd films. The indicated impurity concentrations are those determined by chemical analysis of the ingots from which the films were made.

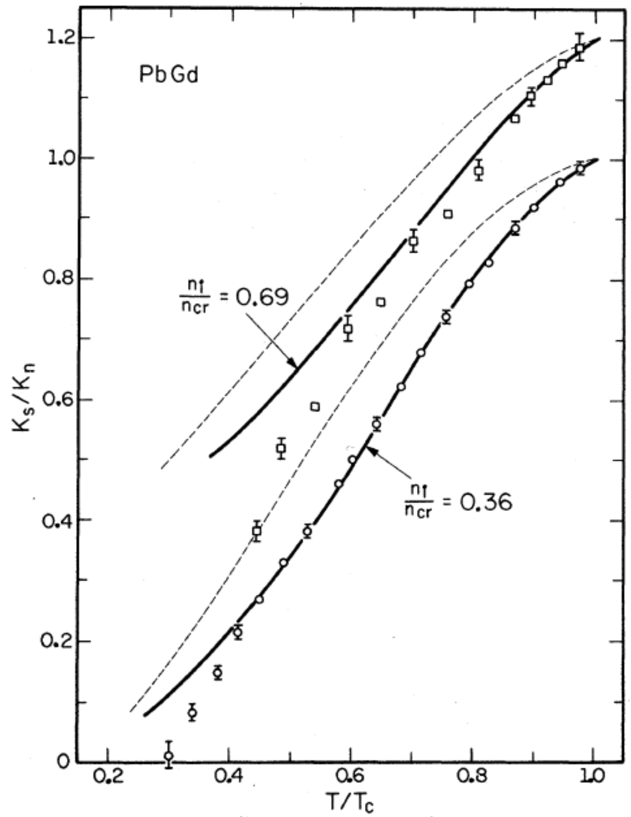


FIG. 6. Reduced thermal conductivity as a function of the reduced temperature for the two Pb-Gd films. The dashed lines show the theoretical values[4,5] for a weak-coupling superconductor, and the solid lines show the values obtained when the theory is modified by strong-coupling corrections for a reduced gap appropriate for pure lead, 4.3. The zero has been shifted up by 0.2 for the higher-concentration alloy to avoid overlap. The indicated impurity concentrations are those determined from the measured transition temperatures.

B.J.Mrstic, D.M.Grinsberg, PRB7, 4844 (1973)

Localized states?

Heat conduction conventional SC: magnetic impurities

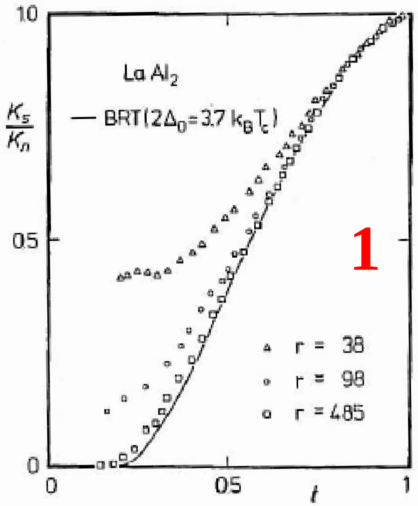


Fig. 4. Reduced thermal conductivity K_s/K_n vs. T/T_c for a $LaAl_2$ single crystal in three different annealing stages, characterized by the resistance ratio r. Solid line: modified BRT curve

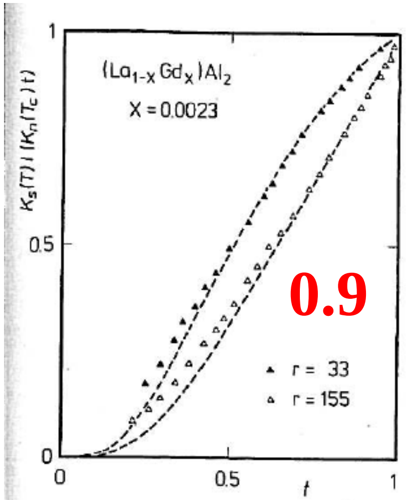


Fig. 5. K_s reduced by $K_n(T_c)\,T/T_c$ as a function of T/T_c for the (La, Gd)Al_2 single crystal with 0.23 a/o Gd before (full triangles) and after annealing (open triangles). Dashed curves: fit results corresponding to Equation 5 with the parameters: $y_0=0.9$, $\tau_1/\tau_2=0.0$, $\tau/\tau_1=0.0$ ($r=33$) and 0.6 ($r=155$)

$CeAl_2$:Gd

T_c/T_{c0}

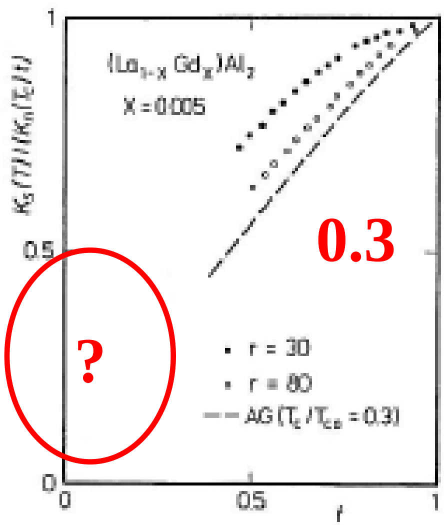


Fig. 9. $K_s/[K_n(T_c)\,t]$ vs. t for the (La, Gd)Al_2 single crystal with 0.5 a/o Gd before (full circles) and after annealing (open circles). Dashed line: Ambegaokar-Griffin theory

Suggestion:

Localized-delocalized character of the quasi-particles

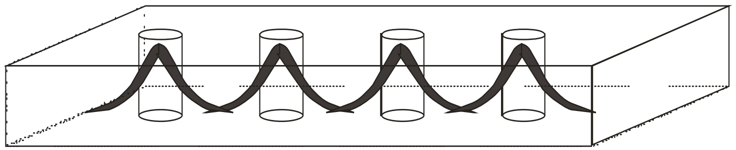
Need to include into consideration

distance of impurity level to the band-gap

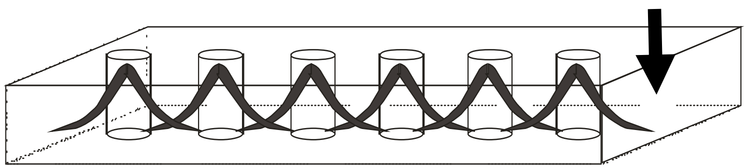
J.H. Moeser, F. Steglich, Z. Physik B25, 339 (1976)

Thermal conductivity of Conventional SC: Vortex State

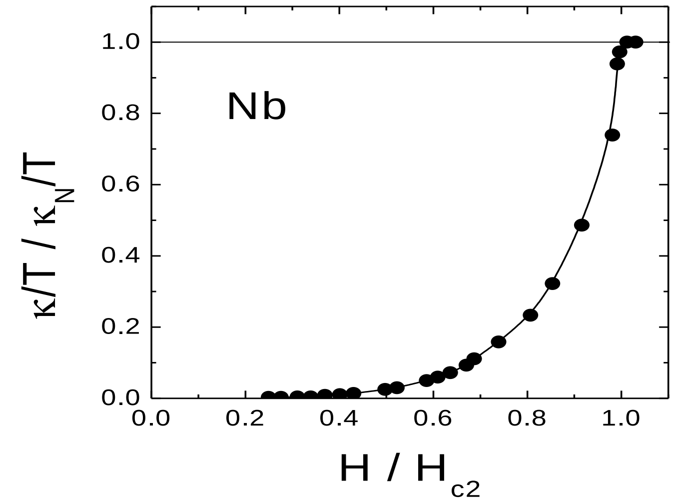
H > H$_{c1}$ Vortex state



H increases



Transport: overlap of the bound states

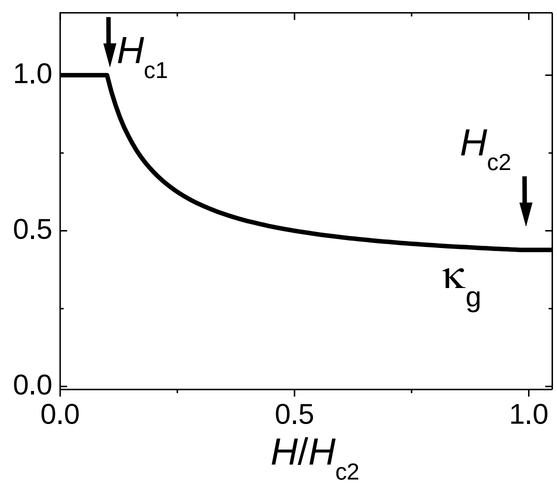


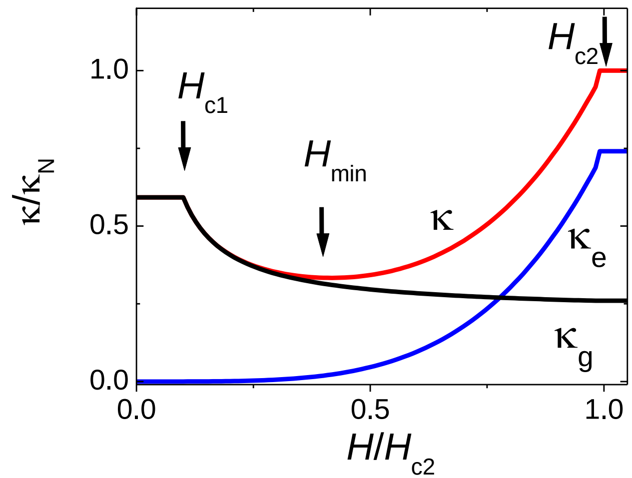
ACTIVATED BEHAVIOR OF THERMAL CONDUCTIVITY

Phonons, $T \neq 0$

$1/\kappa_g \sim H \sim \gamma_S/\gamma_N$

Vortex scattering

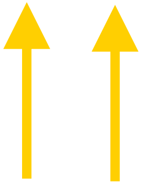




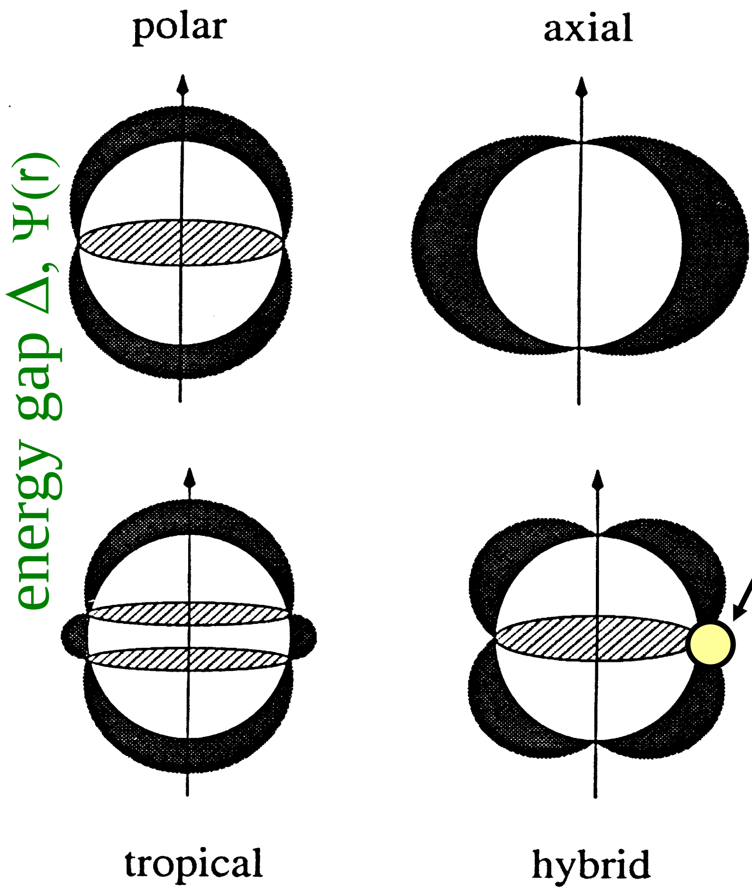
‘Unconventional’ superconductivity

The symmetry of the SC state is lower than the symmetry of the N state

electronic
coupling
mechanism



spin triplet

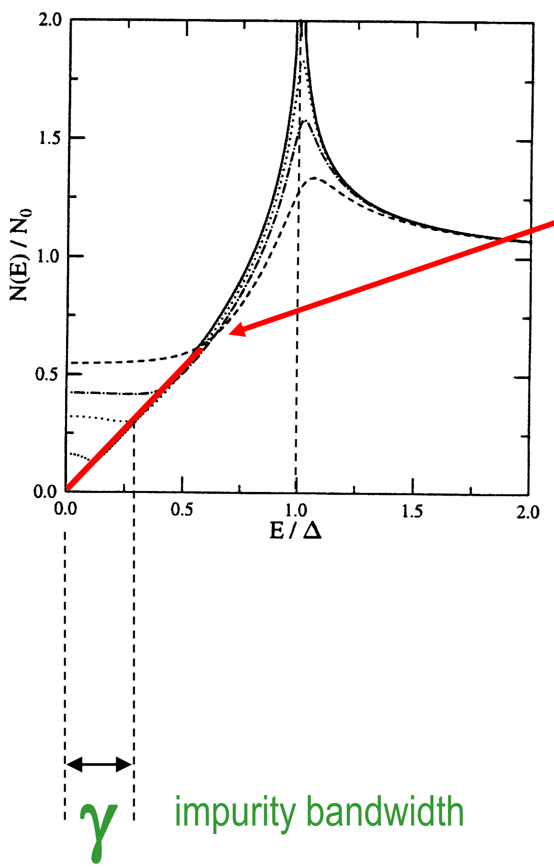


Gap is not constant
at nodes the SC gap vanishes

‘Unconventional’ superconductivity: residual linear term

density of states

$T \ll T_c$



Clean limit

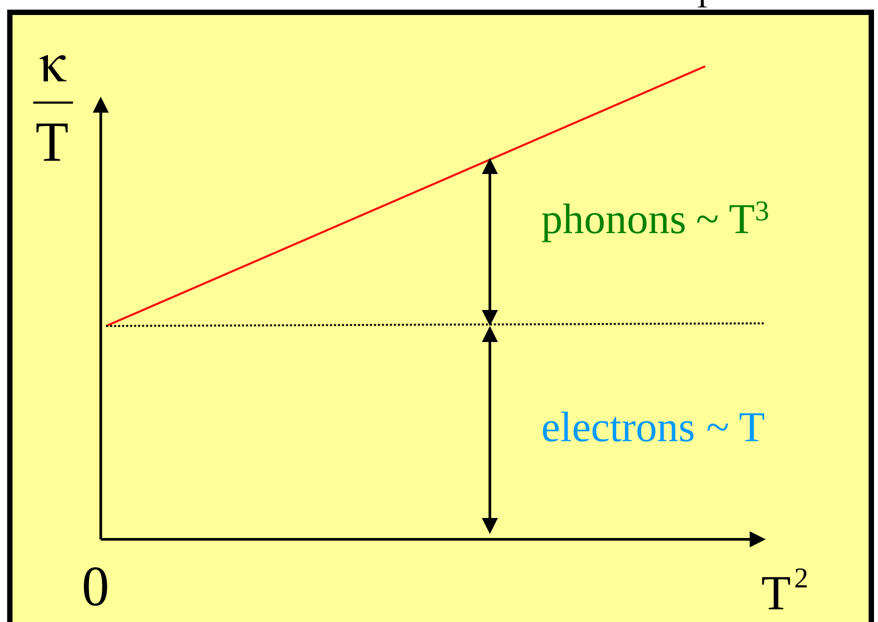
linear increase of density of states with E

Standard techniques

penetration depth $\lambda^{-2} \propto T^{\beta}$

heat capacity $C \propto T^{\gamma}$

NMR relaxation rate $T_1^{-1} \propto T^{\delta}$



No information on location of the nodes on FS

Unconventional SC =bad thermal metal

Heat conduction SC zero field: electrons

Full gap: "Conventional"

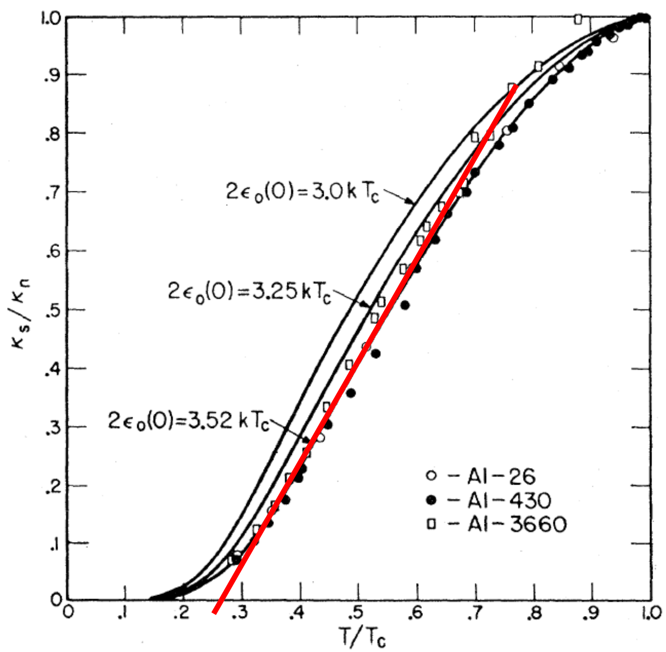
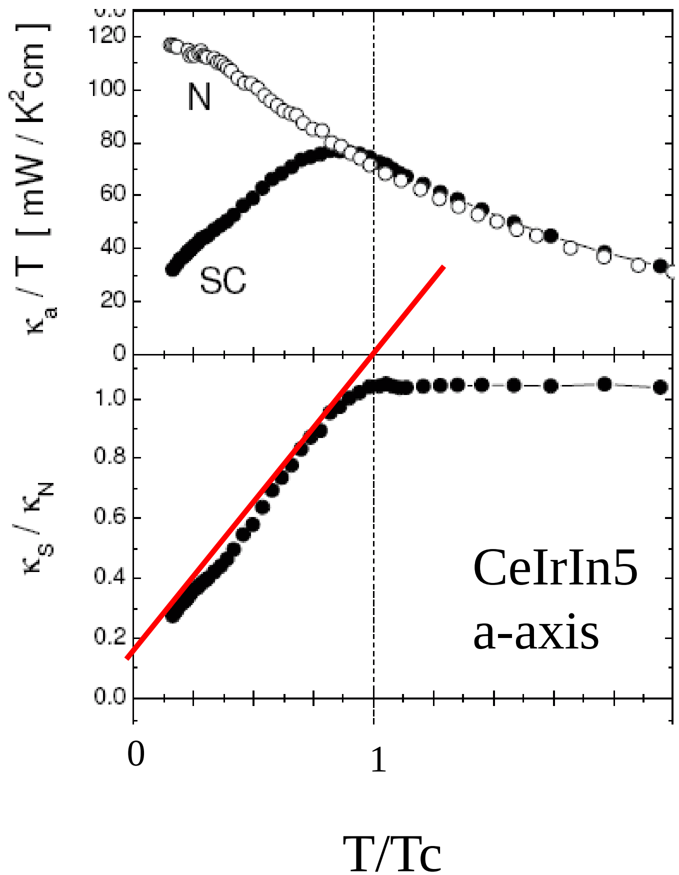


FIG. 3. Ratio of superconducting to normal thermal conductivity for aluminum.

C. B. Satterthwaite, Phys. Rev. **125** 873 (1962)

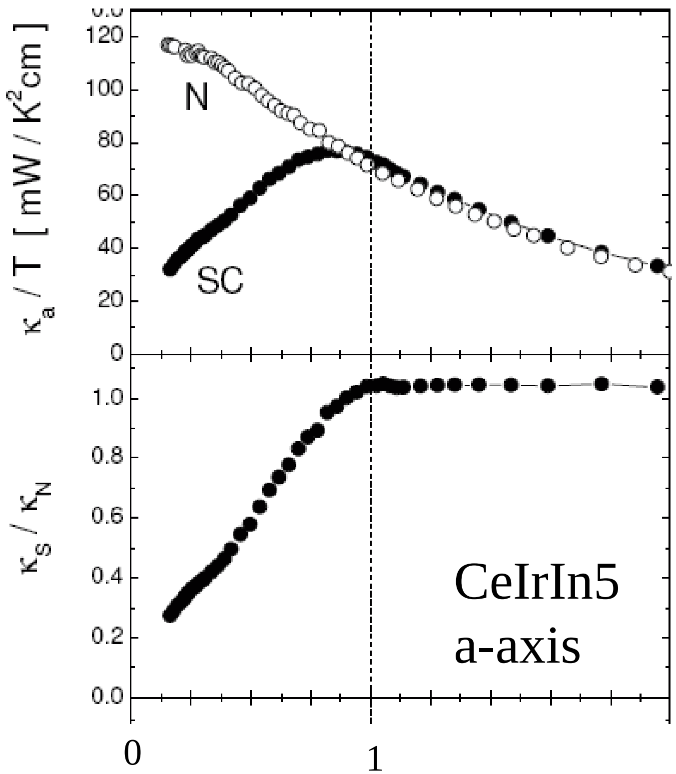
Nodal: "Unconventional"

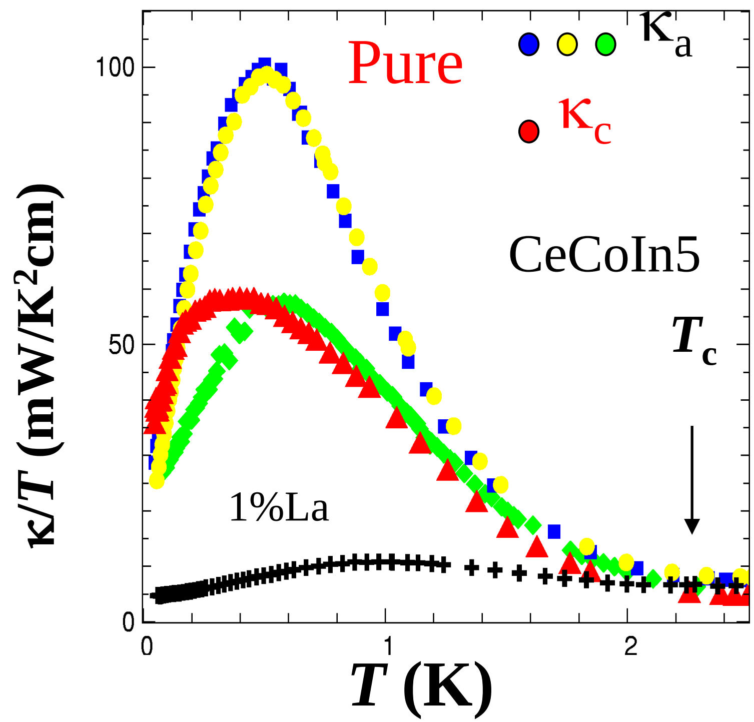


$\rho(Tc)/\rho(0)\sim 1.5$

H.Shakeripour, unpublished

Heat conduction SC zero field: effect of inelastic scattering





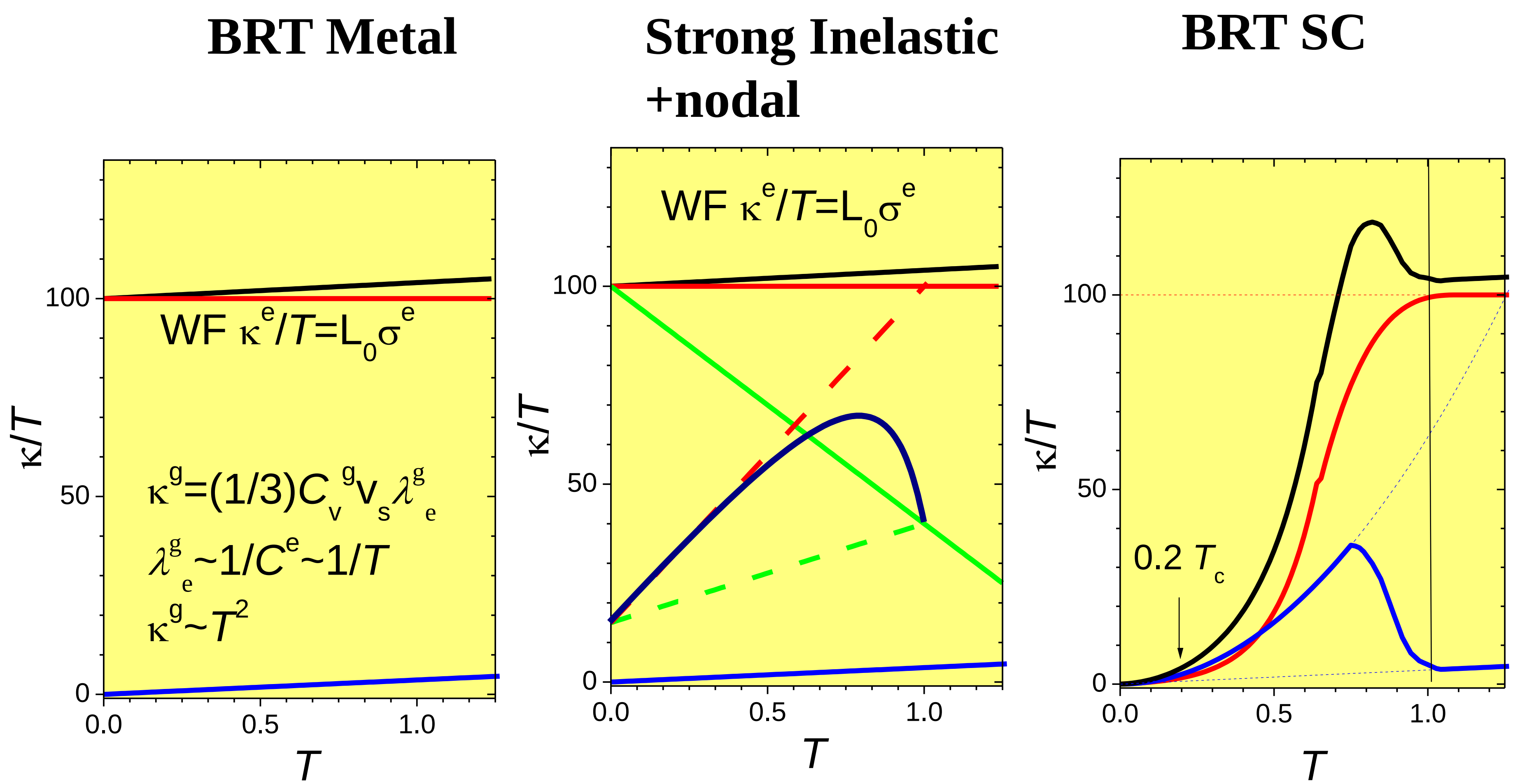
T/Tc

$\rho(Tc)/\rho(0) \sim 1.5$

$\rho(Tc)/\rho(0) \sim 20$ pure
~ 5 1%

Similar effect in the cuprates

The physics of heat conduction: Inelastic scattering

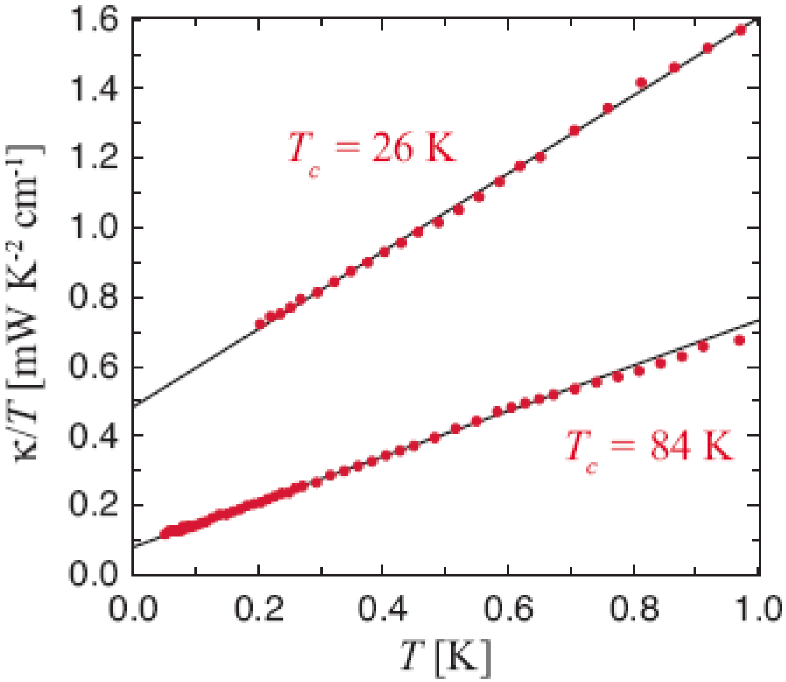


Phonon scattering on conductions electrons dies

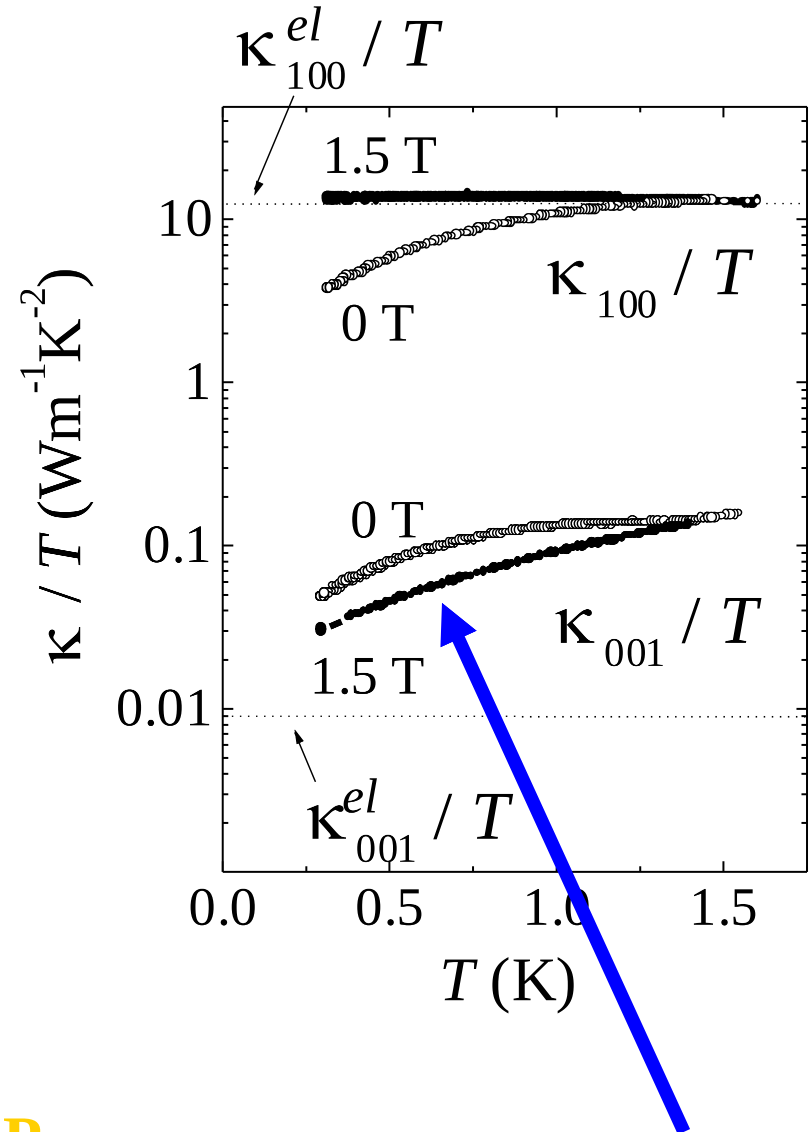
In experiment we study thermal metal-insulator crossover

'Unconventional' superconductivity: residual linear term

Tl2201



Sr_2RuO_4



Residual linear term=
Line nodes in the SC gap

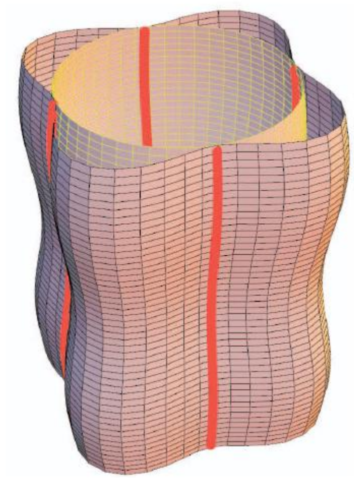
Why κ/T is T-linear?
Electronic contributions+
Phonons are scattered by QP

'Unconventional' superconductivity: anisotropy

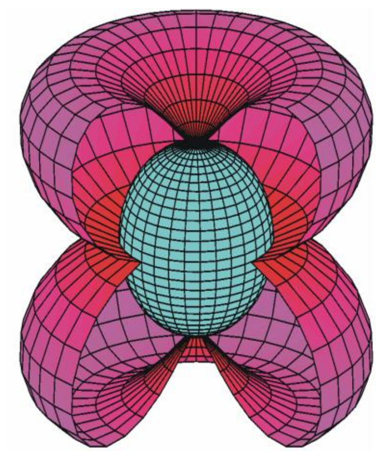
Allowed representations in D_{4h} symmetry

TABLE I: Even-parity (spin-singlet) pair states in a tetragonal crystal with point group D_{4h} [22]. (V = vertical line node, H = horizontal line node.

Representation	Gap	Basis function	Nodes
A_{1g}	s-wave	$1,\ (x^2+y^2),\ z^2$	none
A_{2g}	g-wave	$xy(x^2+y^2)$	V
B_{1g}	$d_{x^2-y^2}$	x^2+y^2	V
B_{2g}	d_{xy}	xy	V
$E_g\ (1,0)$	-	xz	V+H
$E_g\ (1,1)$	-	$(x+y)z$	V+H
$E_g\ (1,i)$	hybrid-I	$(x+iy)z$	H+points



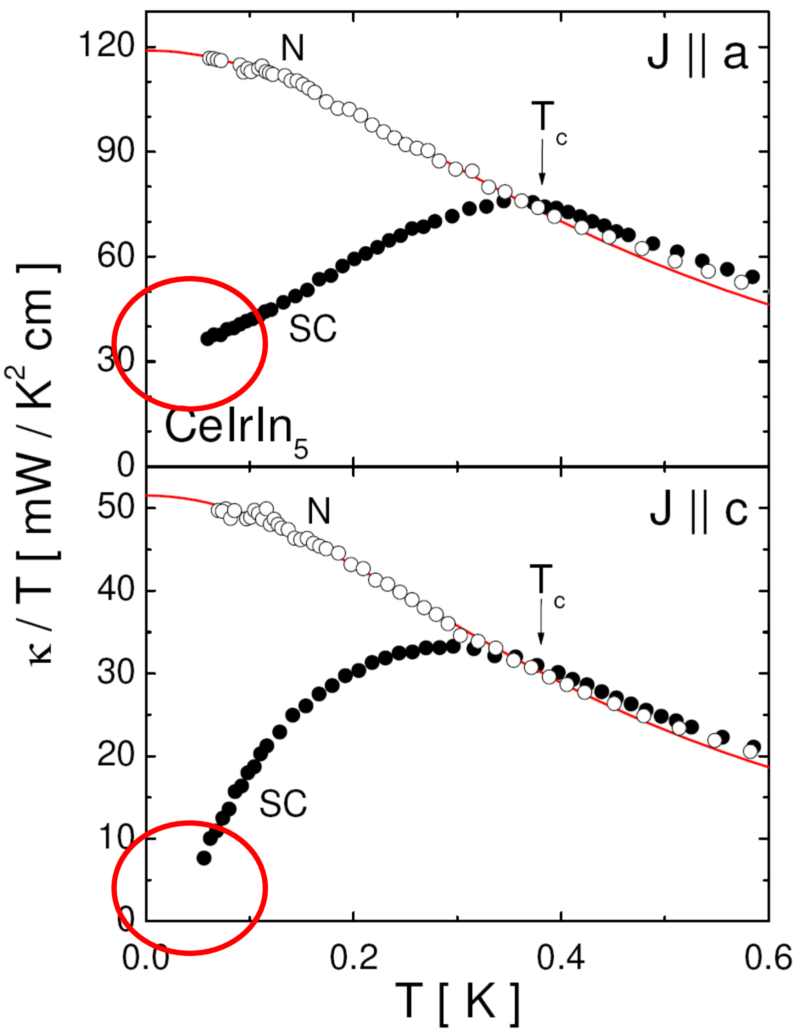
$d_{x^2-y^2}$

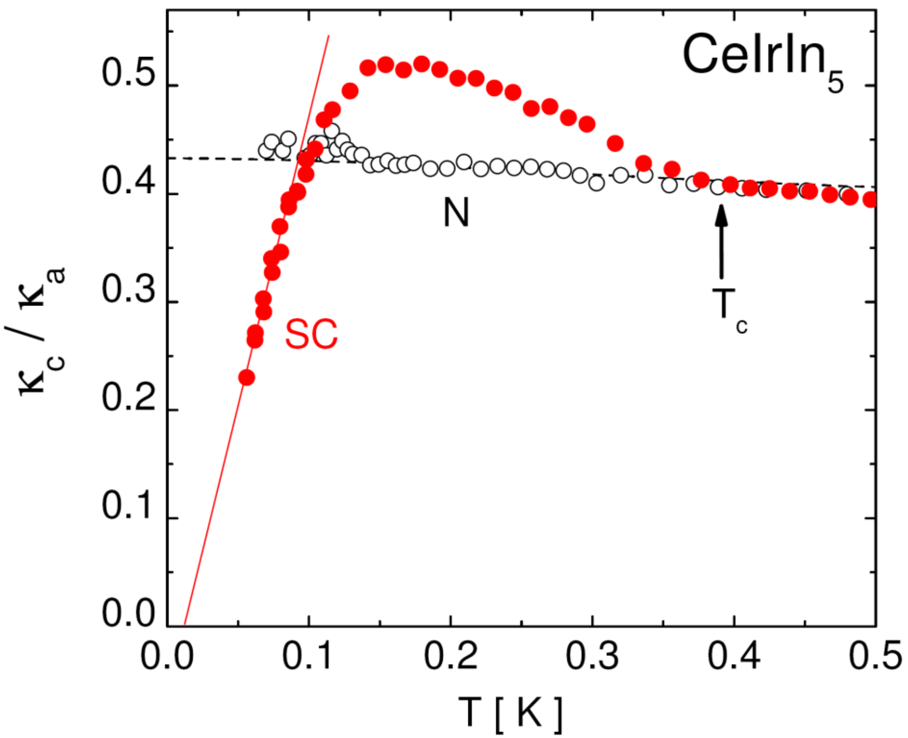


hybrid

$CeIrIn_5$ anisotropic superconductivity

Quasiparticle heat conduction : J // a vs J // c



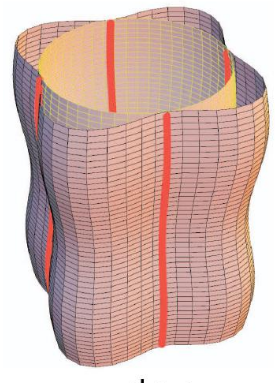


→ pronounced *a-c* anisotropy of nodal structure

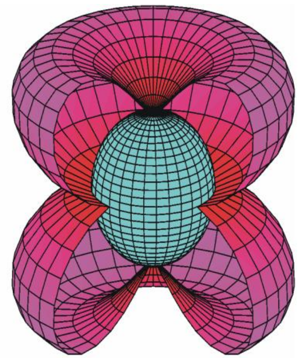
H. Shakeripour *et al.*, PRL 99, 187004 (2006)

$CeIrIn_5$ anisotropic superconductivity

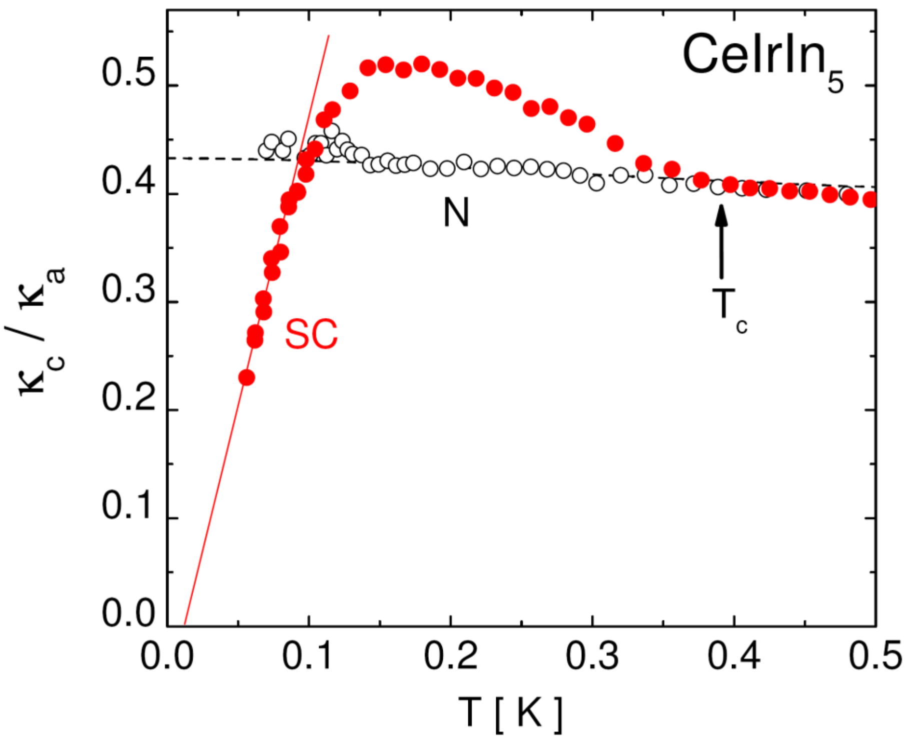
Quasiparticle heat conduction : J // a vs J // c



Vertical line nodes : NO



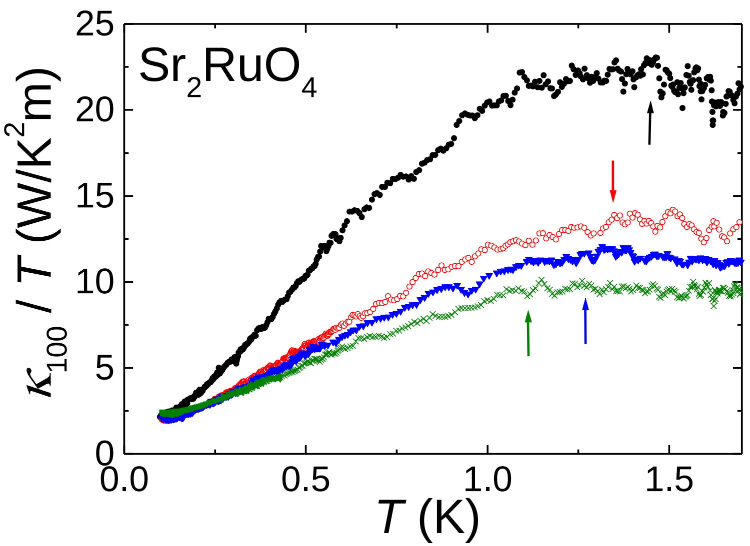
Hybrid-I gap : OK

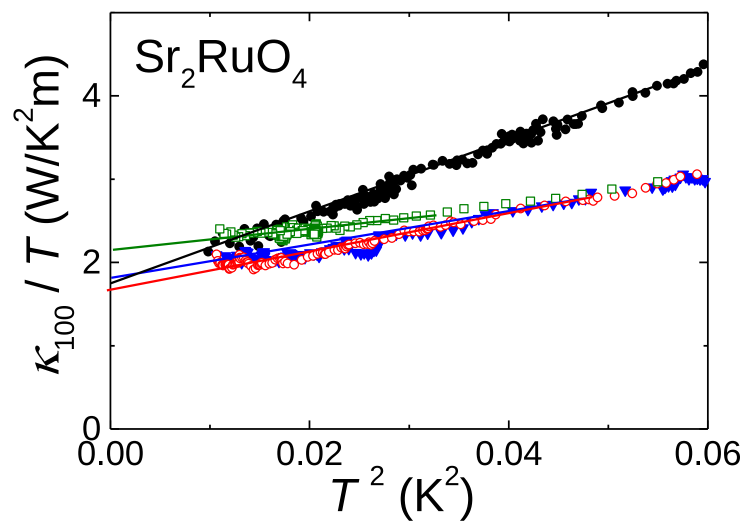


→ E_g (1,i) state

H. Shakeripour *et al.*, PRL 99, 187004 (2006)

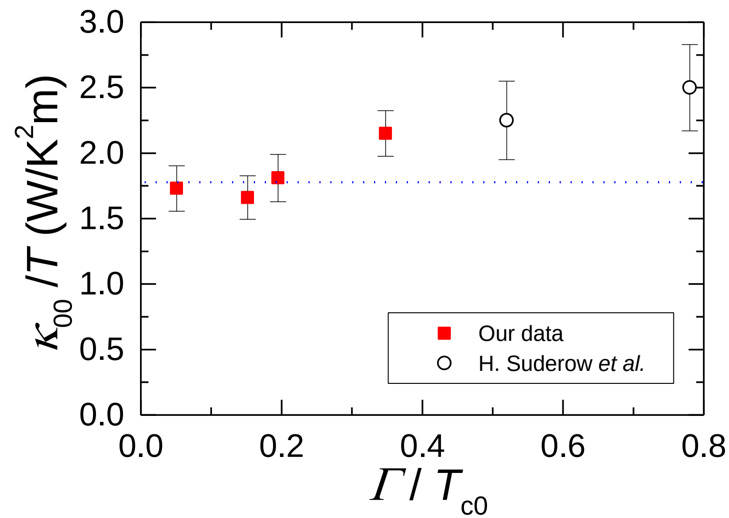
Thermal conductivity: effect of impurities





Clean limit:
Universal conductivity

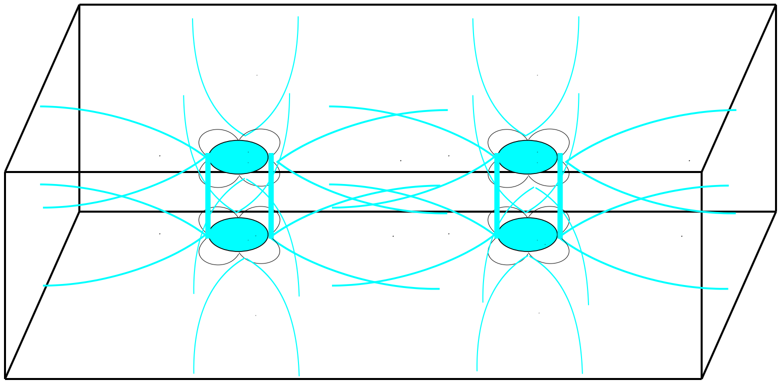
$$\boxed{\frac{\kappa_{00}}{T} = \mathrm{const}(\Gamma)}$$

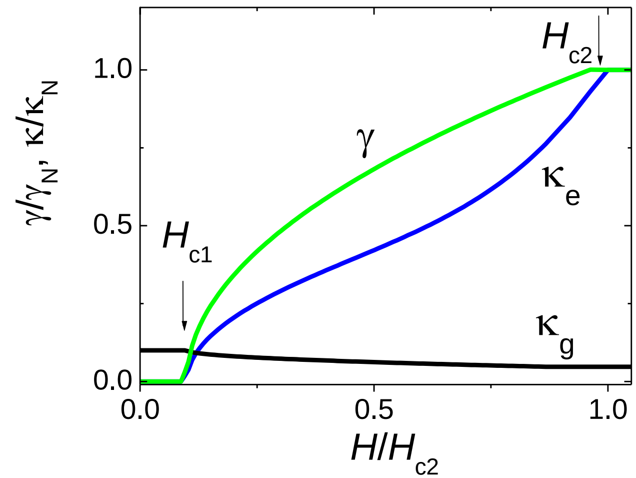


M. Suzuki *et al.*, PRL 88, 227004 (2002)

Thermal conductivity of Unconventional SC: Vortex State

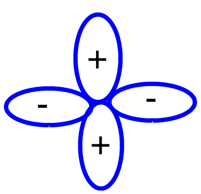
H > H_{c1} Vortex state

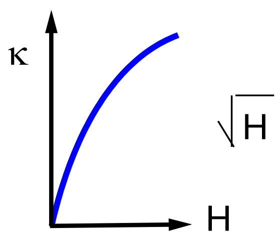




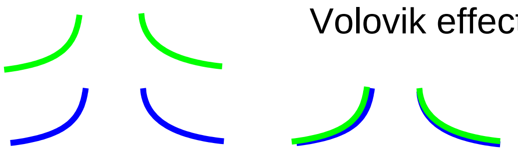
κ: sqrt(*H*) INCREASE WITH FIELD ~H_{c1}

Transport: bulk itinerant states





Volovik effect

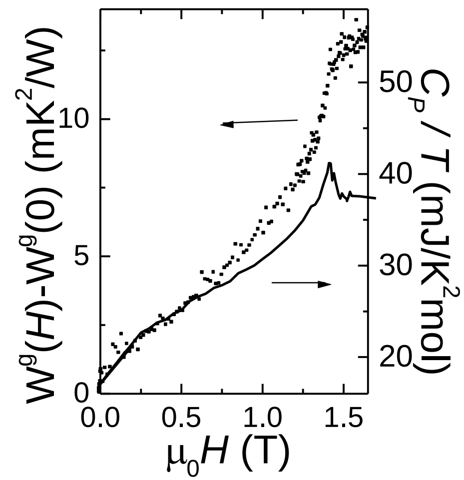


nodal direction

Phonons, $T \neq 0$

$1/\kappa_g \sim \text{sqrt}(H) \sim \gamma_S/\gamma_N$

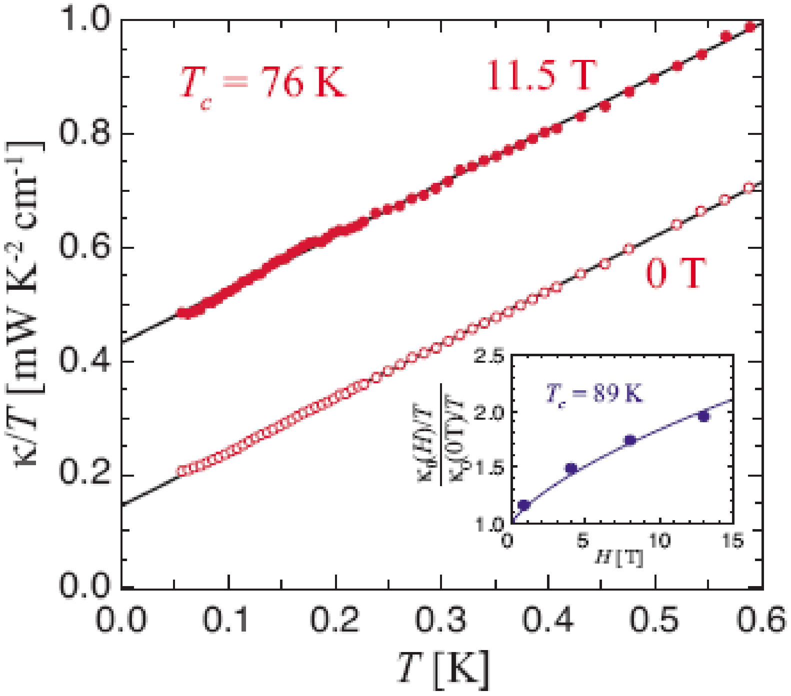
QP scattering

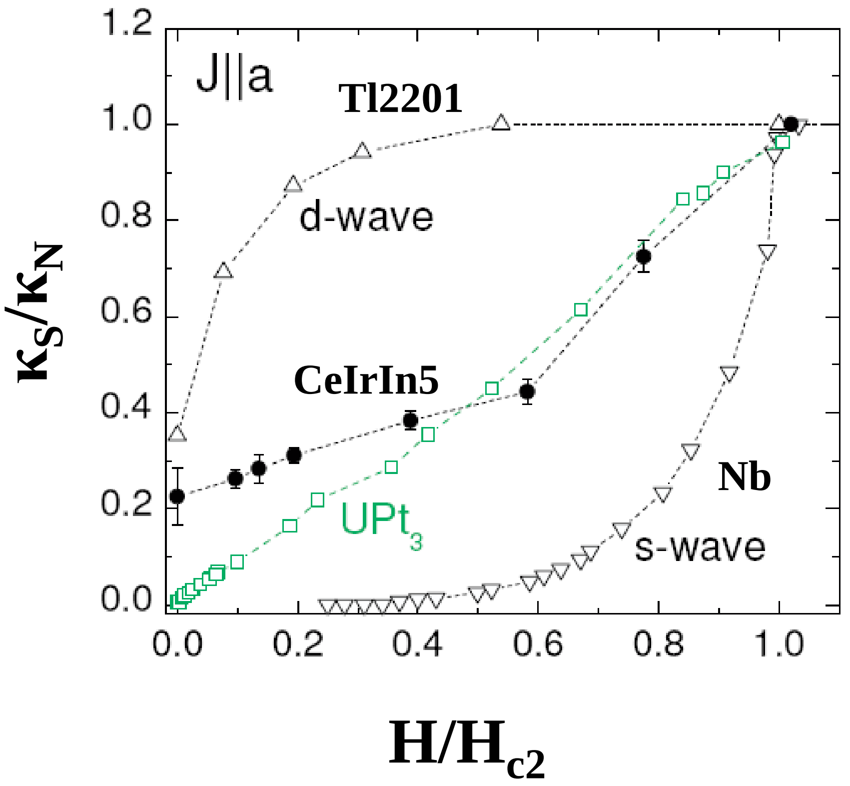


Close relation between the structure of vortex and κ

Thermal conductivity of Unconventional SC: H dependence

Tl2201

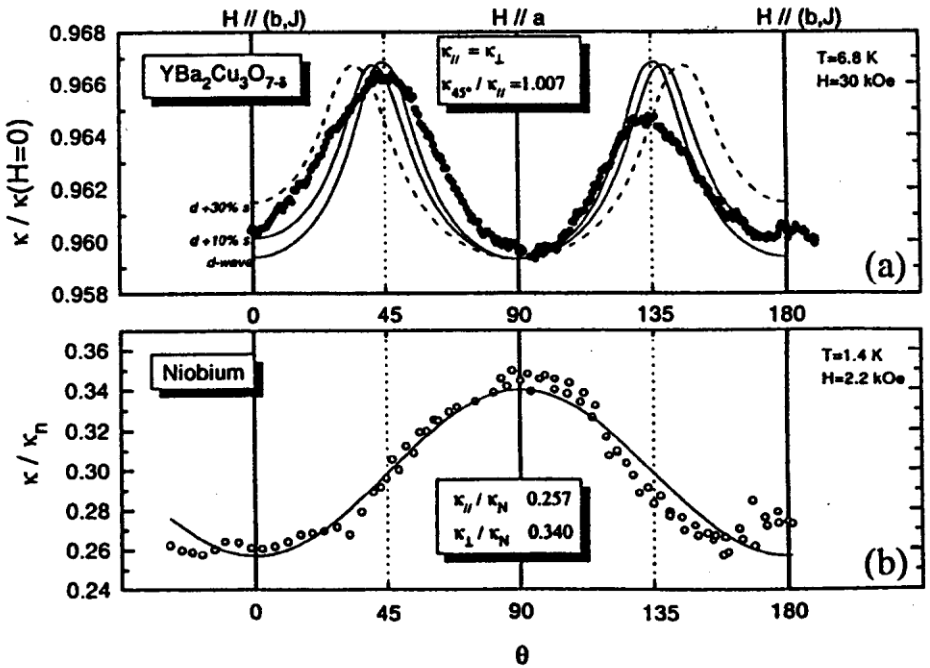




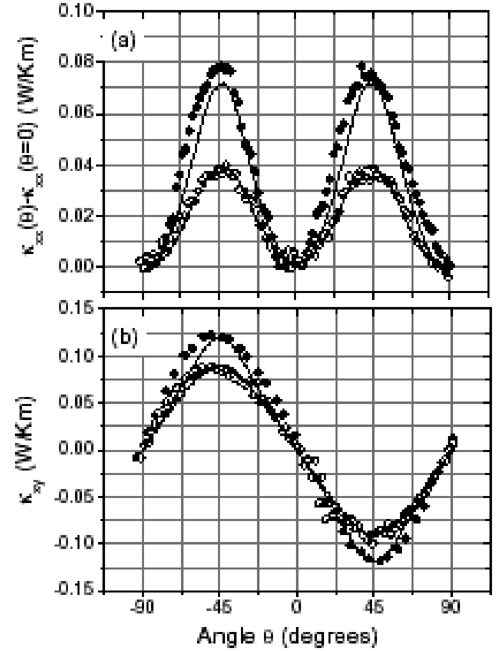
Thermal conductivity with in-plane rotation of H

YBCO

Nb



YBCO



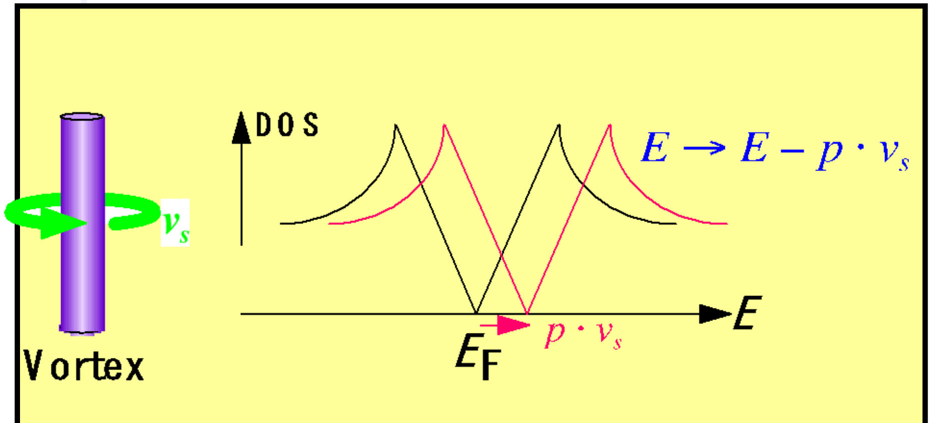
H. Aubin *et al*. PRL 78, 2624 (97)

R.Ocano and P.Esquinazi, cond-mat/0207072

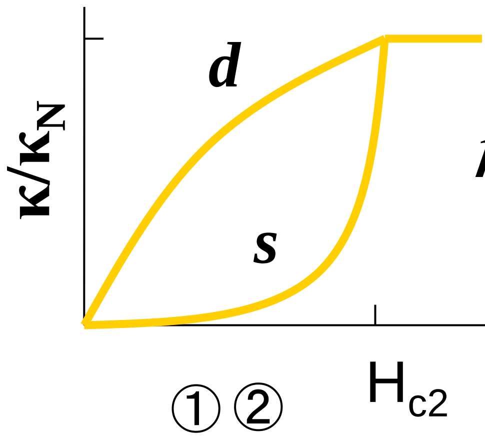
YBCO	Fourfold symmetry
Nb	No fourfold symmetry
	Twofold symmetry

The difference between the effective DOS for QPs traveling parallel to the vortices and for those moving in the perpendicular direction

Doppler shift of the quasiparticle spectrum

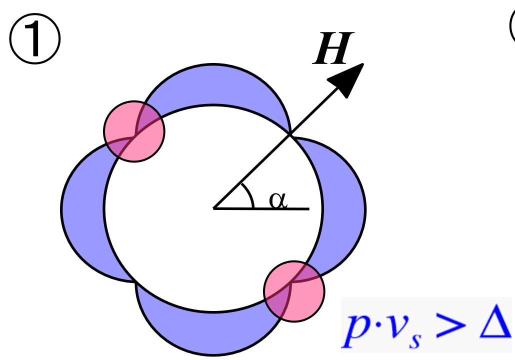


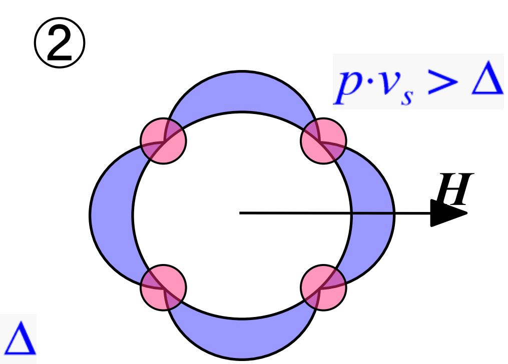
G.E.Volovik, JETP Lett. 58, 469 (1993)

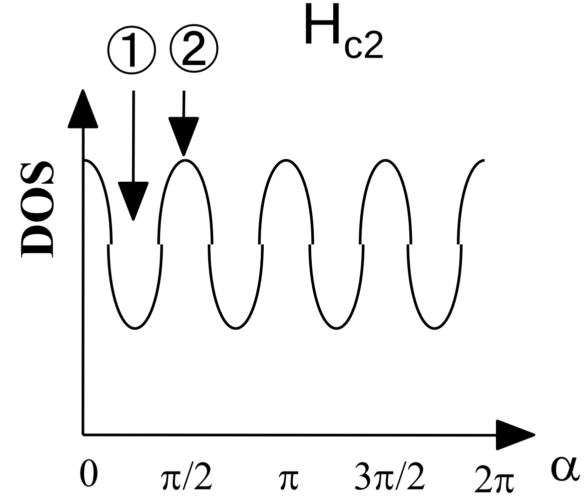


$$\kappa \sim \sqrt{H}$$

$d_{x^2-y^2}$ – case







2 nodes contribute
DOS small

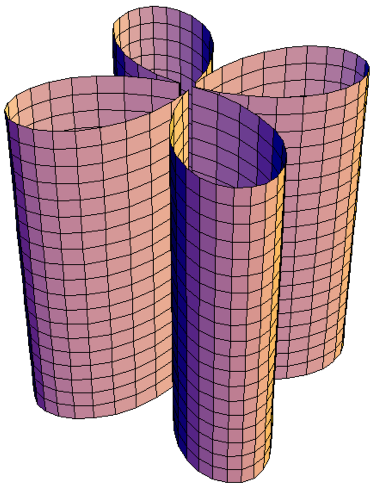
4 nodes contribute
DOS large

4-fold oscillation of DOS

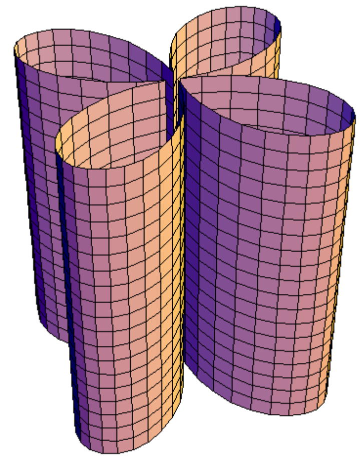
I.Vekhter *et al*. PRB 59, R9023 (99)
H.Won and K. Maki, cond-mat/0004105

4-fold oscillation of QP scattering time

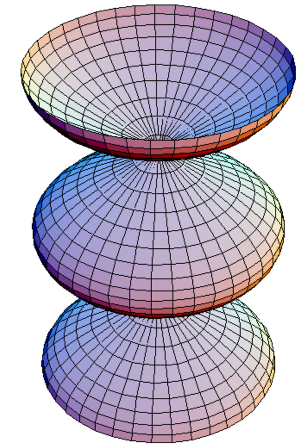
$CeCoIn_5$

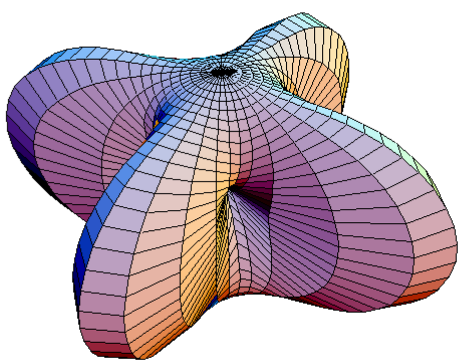


κ-$(ET)_2Cu(NCS)_2$

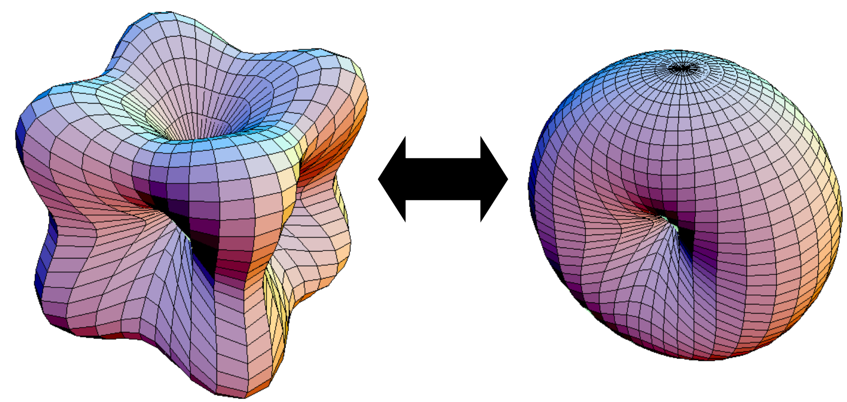


Sr_2RuO_4





YNi_2B_2C



$PrOs_4Sb_{12}$

Thermal conductivity: phase transition at Hc2

Sr2RuO4

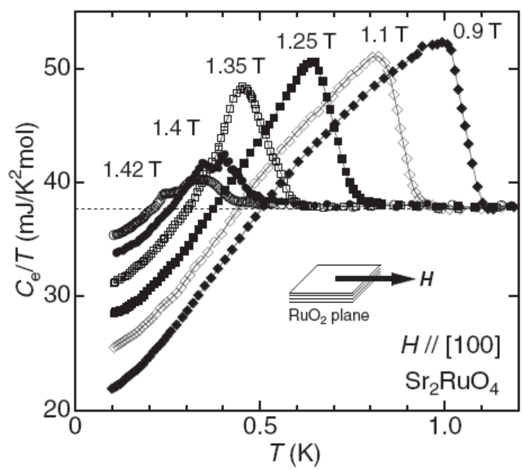


Fig. 1. Temperature dependence of C_e/T in magnetic fields precisely parallel to the [100] direction.

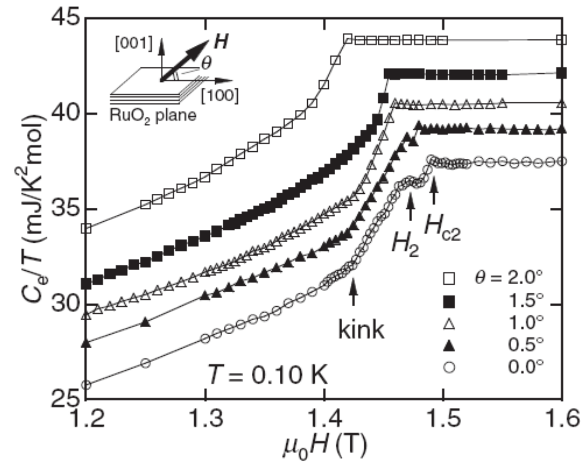


Fig. 3. Transformation of the field dependence of C_e/T near H_{c2} at 0.10 K on each field angle θ. Except for 0.0°, each trace has an offset.

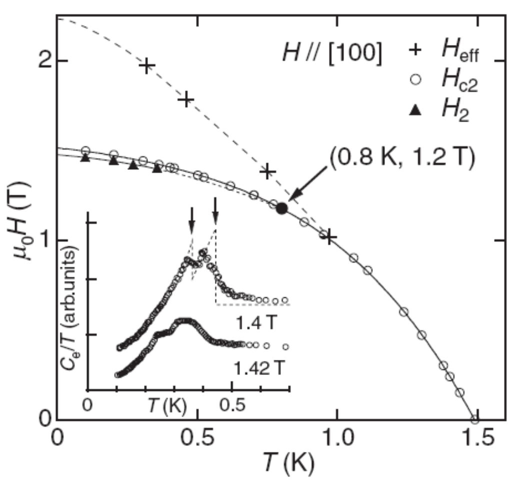


Fig. 2. Phase diagram of Sr_2RuO_4 for $H \parallel$ [100], based on specific heat. H_{c2} and H_2 are the upper critical field and the critical field for the second superconducting transition. H_{eff} is the critical field for normalization shown in Fig. 4(b). The inset shows an enlargement of Fig. 1 and the definition of H_{c2} and H_2.

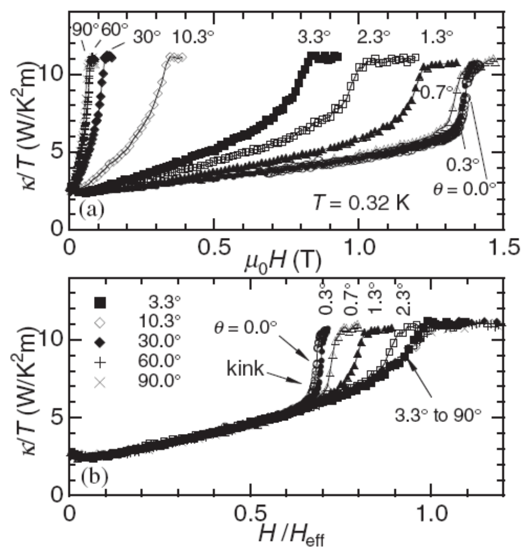
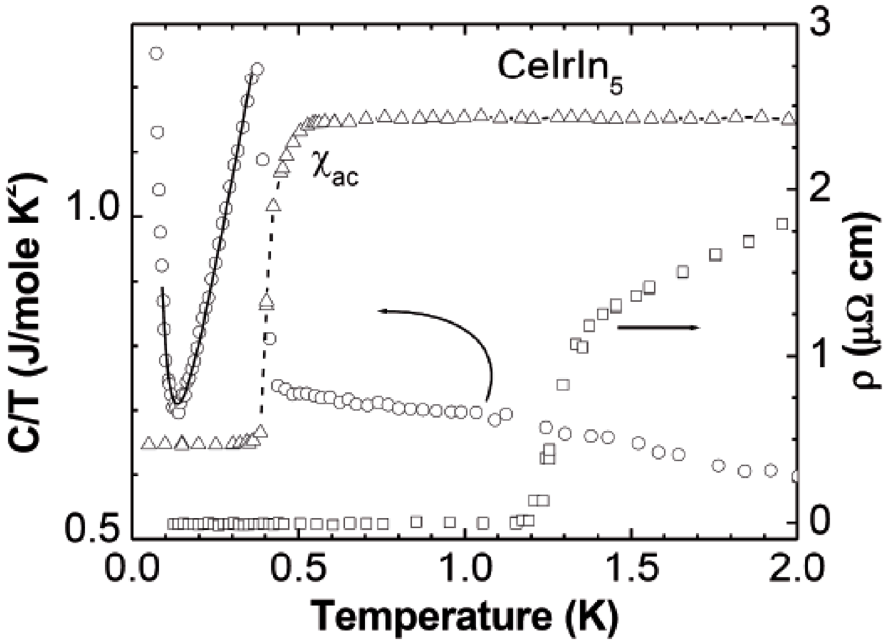


Fig. 4. (a) Transformation of the field dependence of κ/T at 0.32 K on each field angle θ. (b) The same dependence normalized by H_{eff}, treated as a fitting parameter.

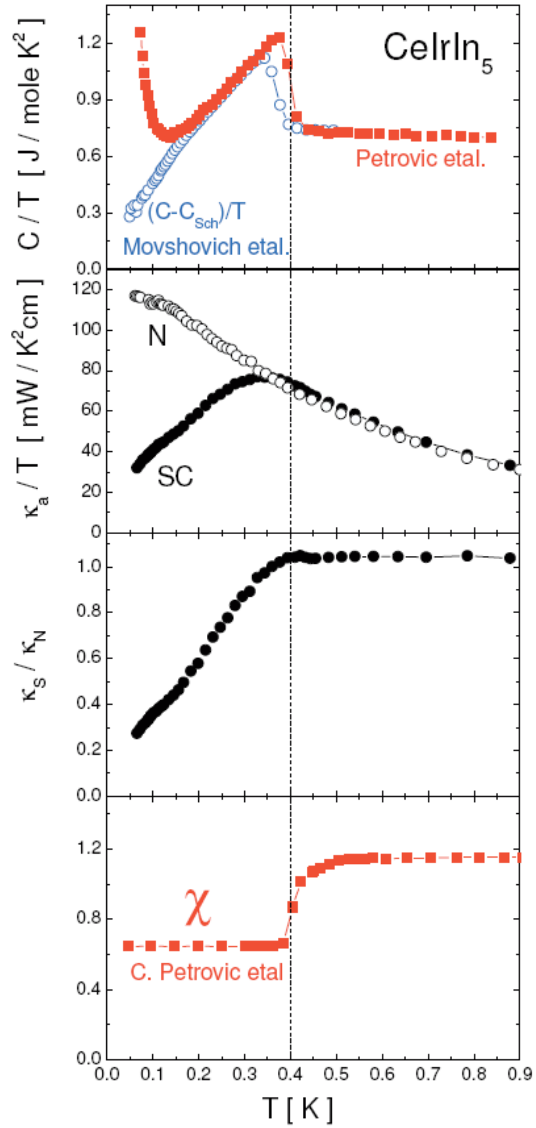
Thermal conductivity of SC: summary

	Isotropic SC	Nodal (Unconventional)
T	$\kappa/T = 0$ in $T=0$ limit	residual linear term
X	no effect	universal
H	activated	immediate increase above H_{c1}
$H\varphi\theta$	2-fold	+represents nodal structure
$J\varphi\theta$	reflects band structure	+represents nodal structure

Thermal conductivity of SC: comparison with other experiments



Petrovic et al.



Advantages of thermal conductivity in the SC state

$$\kappa_e = C_e l_e V_F$$

Thermal conductivity is closely linked with specific heat

Bulk, insensitive to superconducting filaments
No localized contributions (Schottky anomaly)
Insensitive to the transformations in the vortex lattice.
Line nodes (from temperature dependence in low temperature limit)
Position of nodes on the Fermi surface
(dependence on direction of magnetic field and of the heat flow).
Characterization of the upper critical field,
allows discrimination of 1st and 2nd order transitions.

Thermal conductivity	**Specific heat**
Good at T $\rightarrow 0$	**Good at Tc**
Bad at T_c	**Bad at T $\rightarrow 0$**

Heat conduction SC: multiband

MgB2

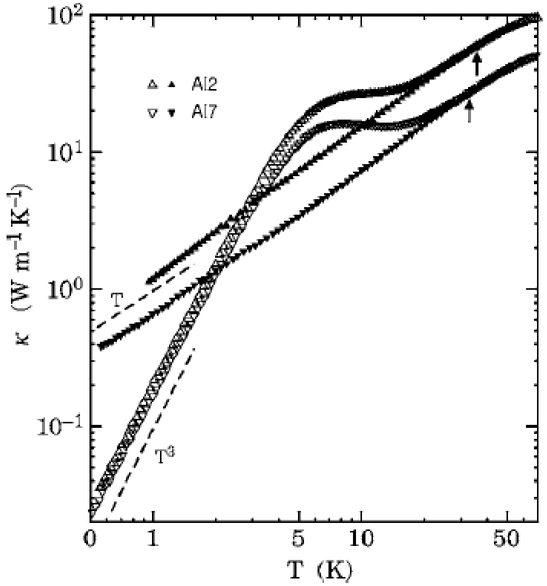
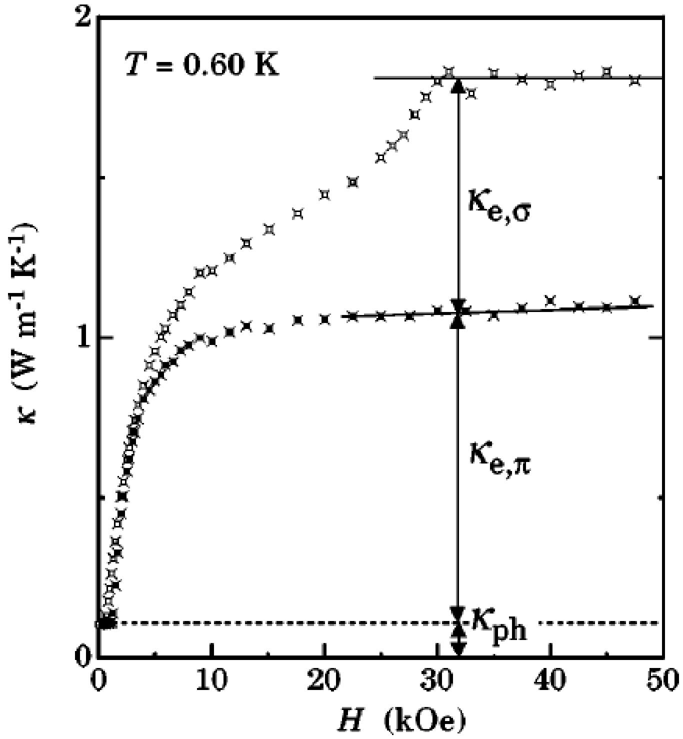
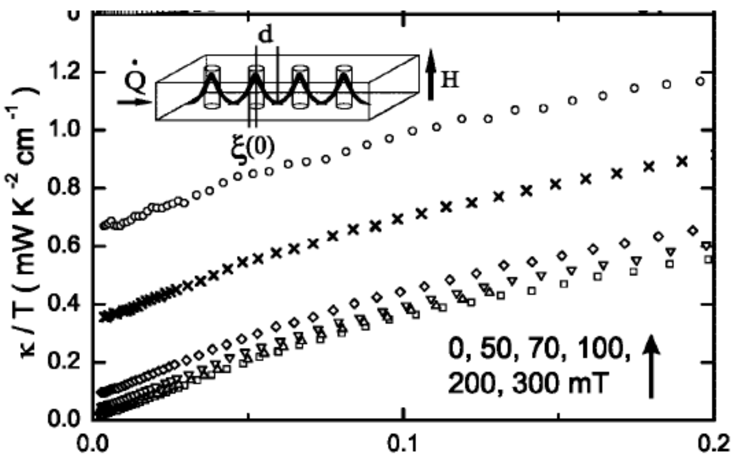


FIG. 1. Thermal conductivity vs temperature in the *ab* plane of single-crystalline $Mg_{1-y}Al_yB_2$ (y=0.02 and 0.07) in zero magnetic field (open symbols) and $H \| c$=50 kOe (solid symbols). The arrows indicate the critical temperatures in zero magnetic field.



Heat conduction SC: multiband

NbSe2



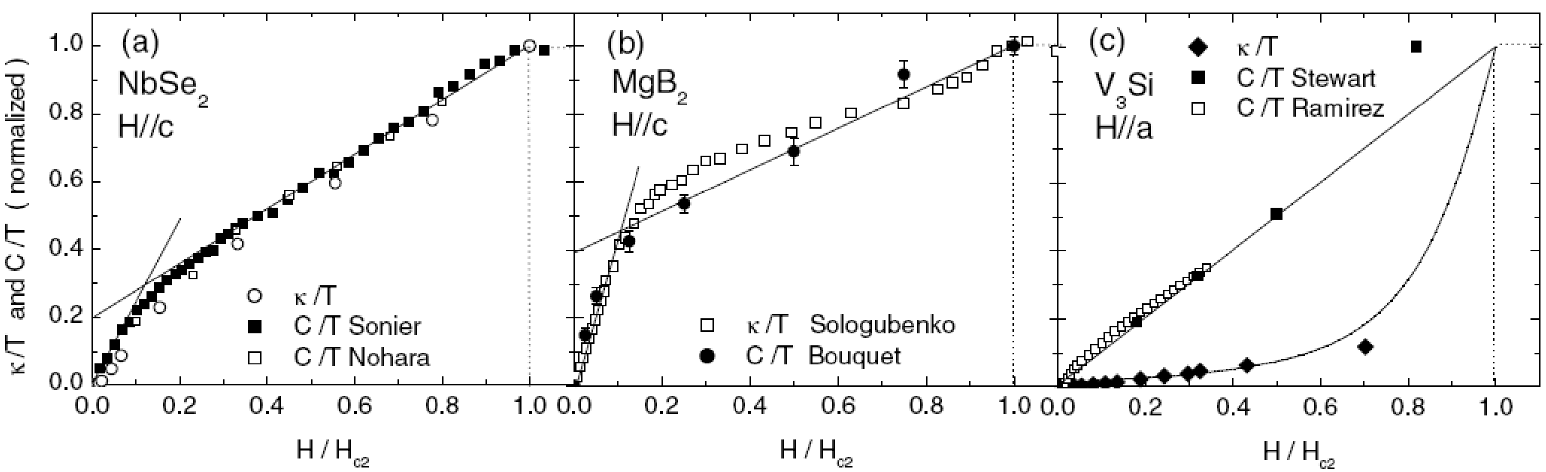
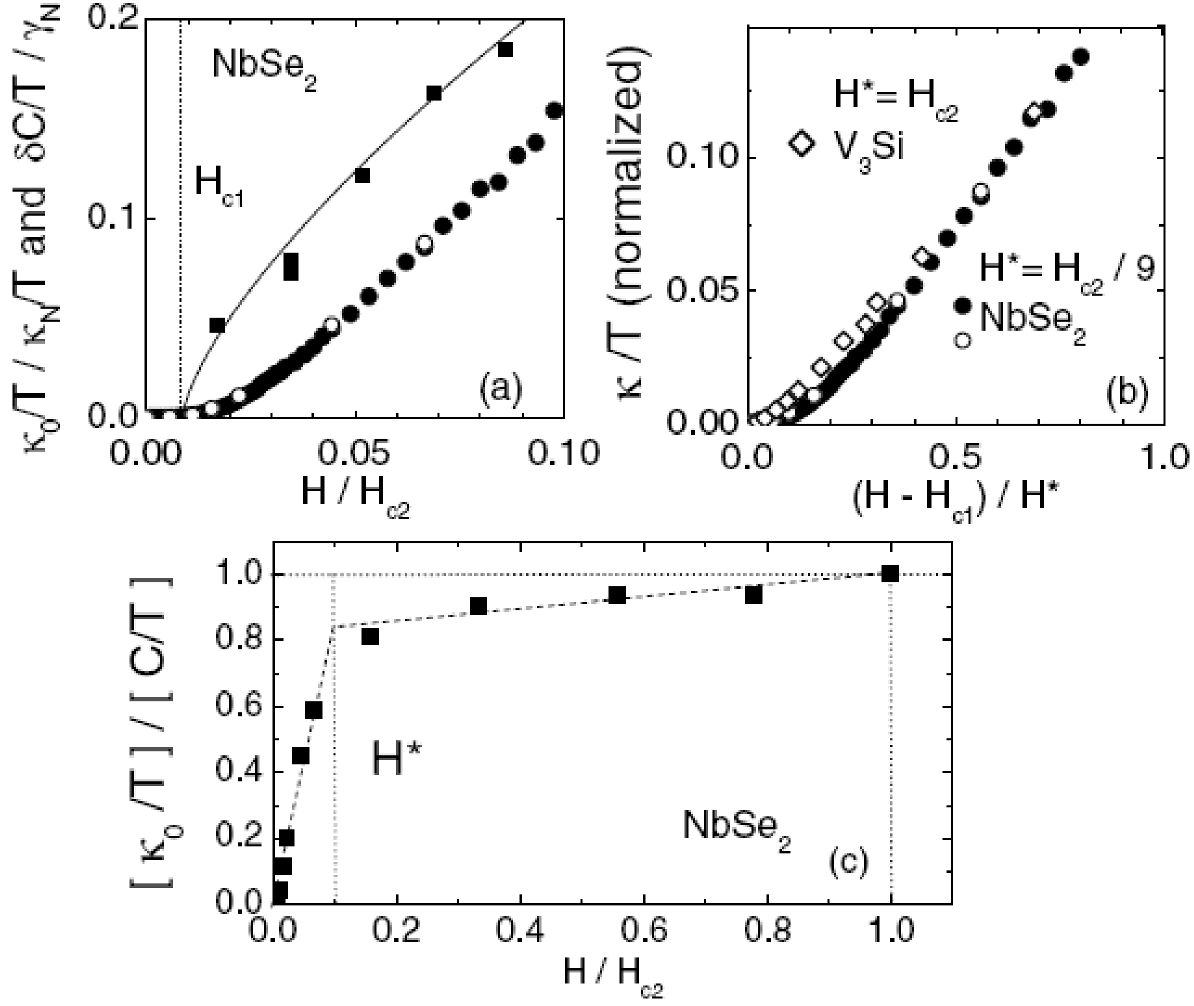


FIG. 3. (a) Thermal conductivity and heat capacity of $NbSe_2$ normalized to the normal state value vs H/H_{c2}. The heat capacity was measured in two different ways: (i) at $T = 2.4$ K on the same crystals as used in this study [10], and (ii) extrapolated to $T \rightarrow 0$ from various temperature sweeps on different crystals [18]. (b) Equivalent data for MgB_2 single crystals [19,20]. (c) Equivalent data for V_3Si, with a theoretical curve for κ/T [9]. The specific heat is measured at $T = 3.5$ K [21] and extrapolated to $T = 0$ [22]. The straight line is a linear fit. The thermal conductivity is seen to follow the specific heat very closely for both $NbSe_2$ and the multiband superconductor MgB_2. It does not, however, for the conventional s-wave superconductor V_3Si.

Heat conduction SC: multiband

NbSe2



Something about quantum criticality

Thermal conductivity of normal metals

Temperature dependence of Lorenz ratio
Scattering processes
Magnetic scattering

Thermal conductivity at QCP

Critical scattering anisotropy
Q-vector of magnetic fluctuations

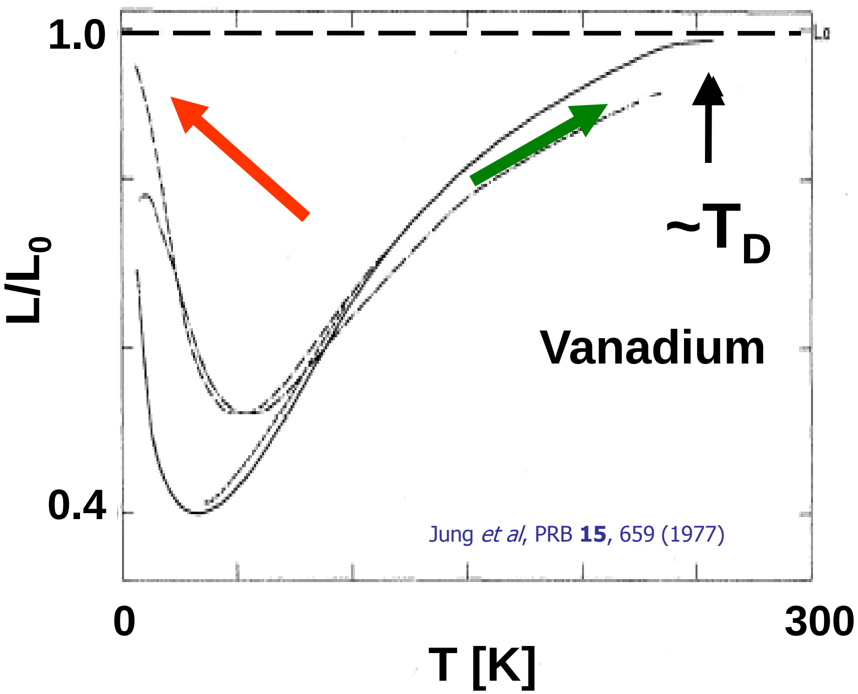
Summary & Conclusions

Temperature dependence of Lorenz ratio

Lorenz ratio $\kappa / \sigma T$

$T \rightarrow 0: \; L = L_0$

Wiedemann-Franz law

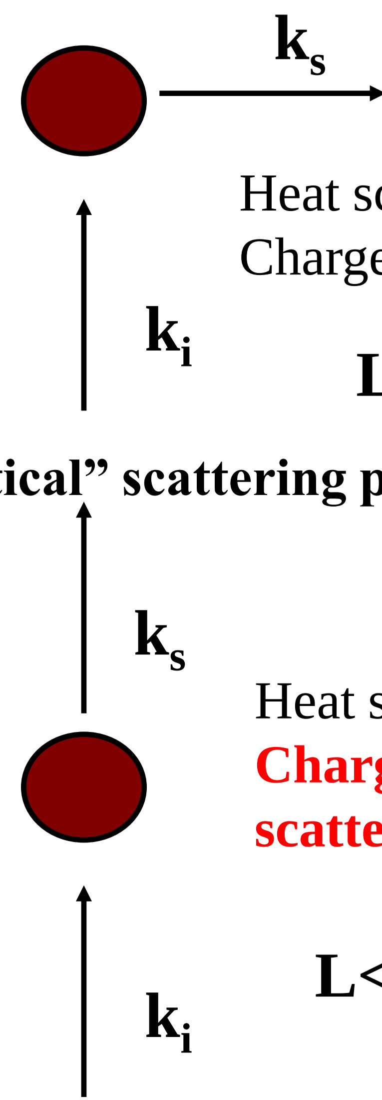


$L \rightarrow L_0$

at $T \sim T_D$ phonon scattering becomes quasi-elastic

characteristic energy scale

"Horizontal" scattering processes



Heat scattered
Charge scattered

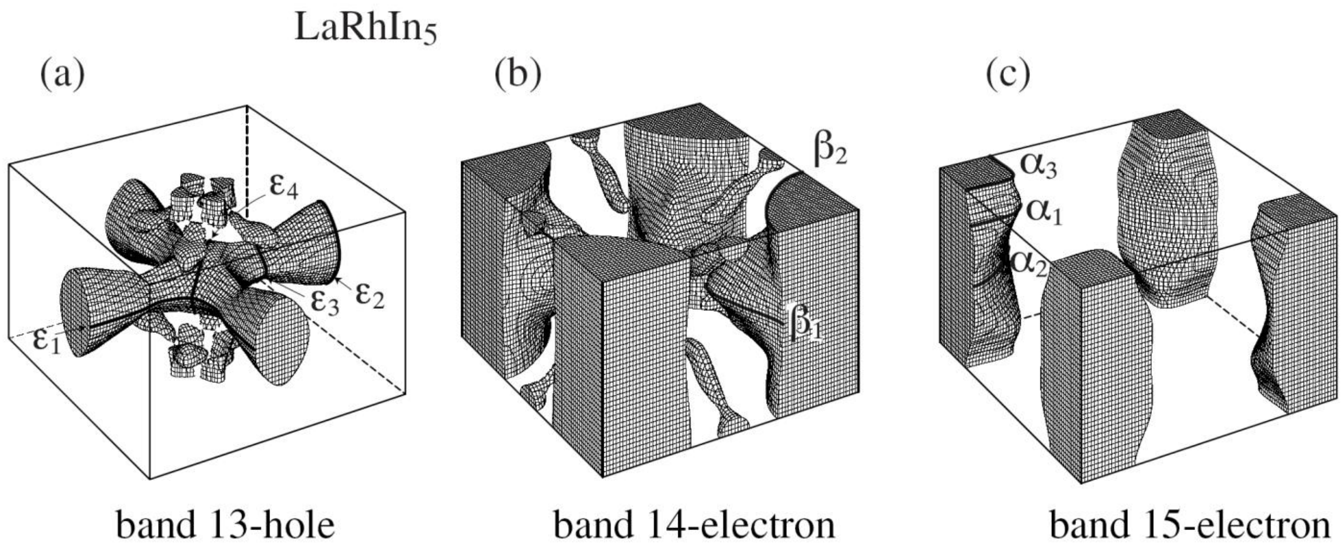
$L = L_0$

"Vertical" scattering processes

Heat scattered
Charge is not scattered

$L < L_0$

$CeRhIn_5$: localized f-electron AF



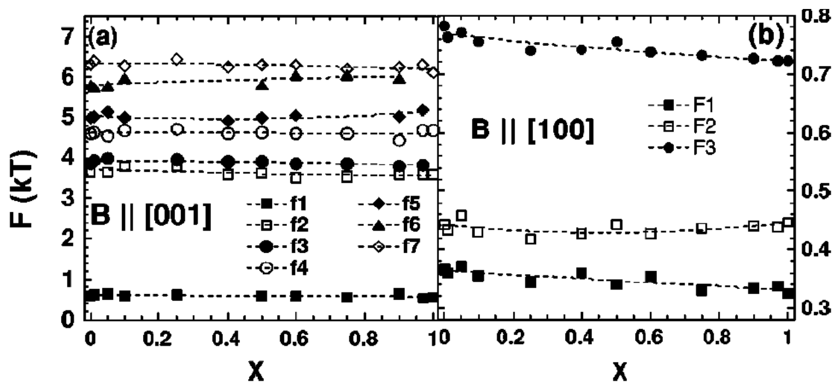
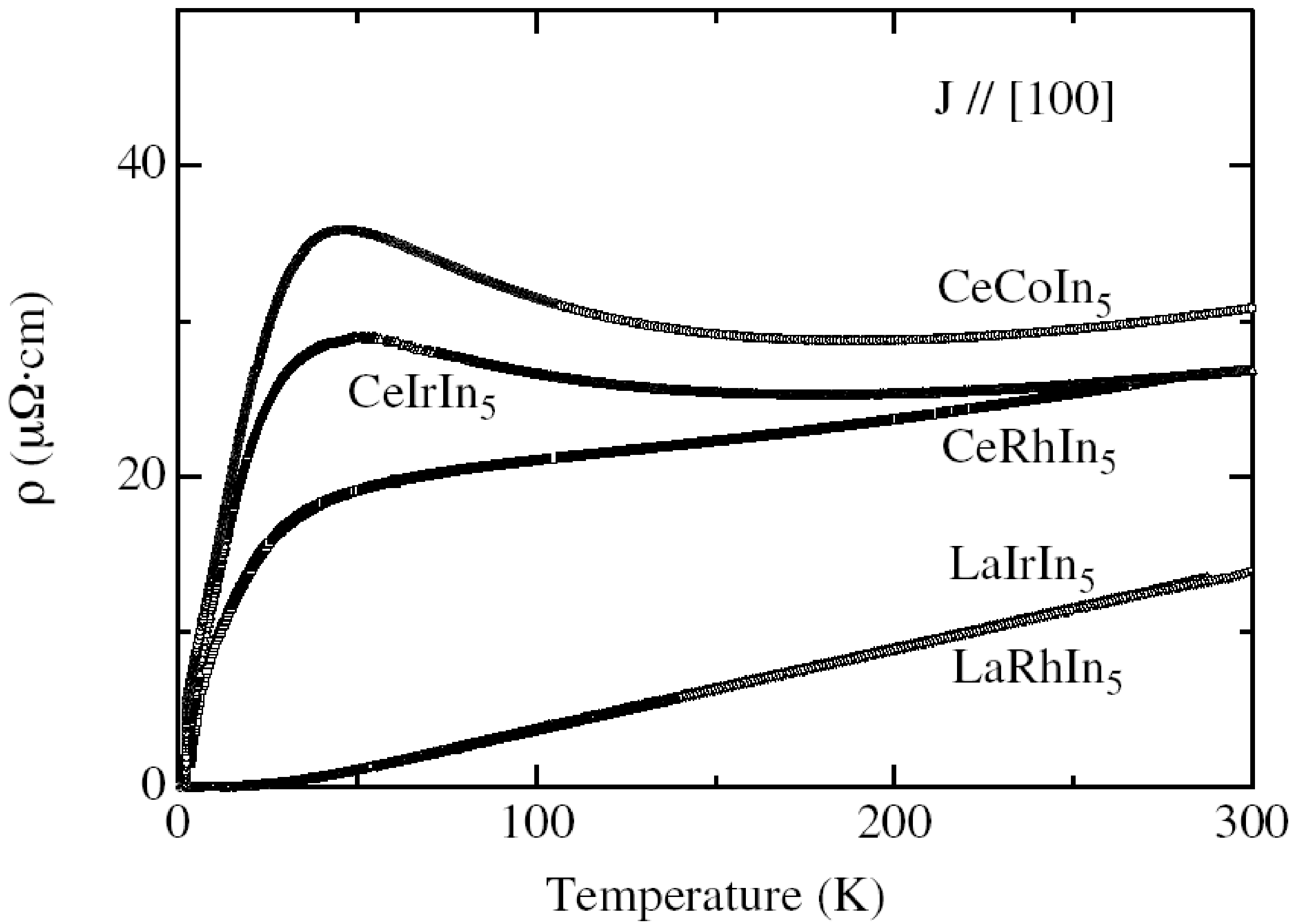


FIG. 2. The principal dHvA frequencies in $Ce_xLa_{1-x}RhIn_5$ plotted versus x, (a) for B applied along [001] and (b) for B applied along [100].

Localized f-electrons

$CeMIn_5$: Magnetic contribution to resistivity



Residual resistivity ρ_0
$LaRhIn_5$ etc. <0.01 μΩ.cm
$CeRhIn_5$ 0.02 μΩ.cm
CeCoIn5 0.1-0.2 μΩ.cm

Importance of purity

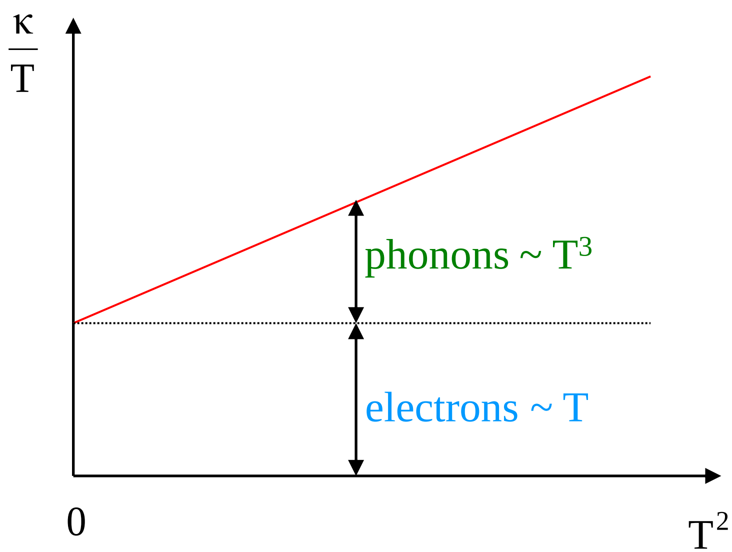
beat the phonons

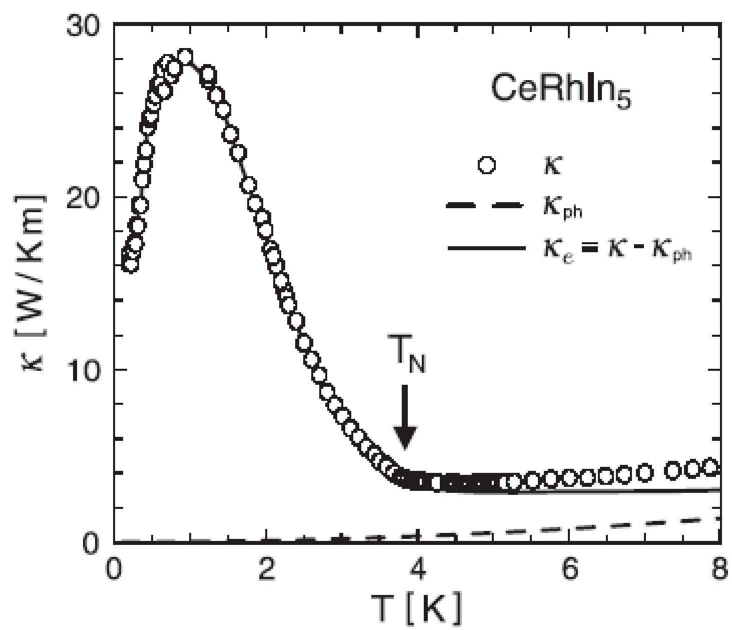
Heat transport

- *Electronic* thermal conductivity

$\rho_0 = 40\ \mathrm{n\Omega\ cm}$

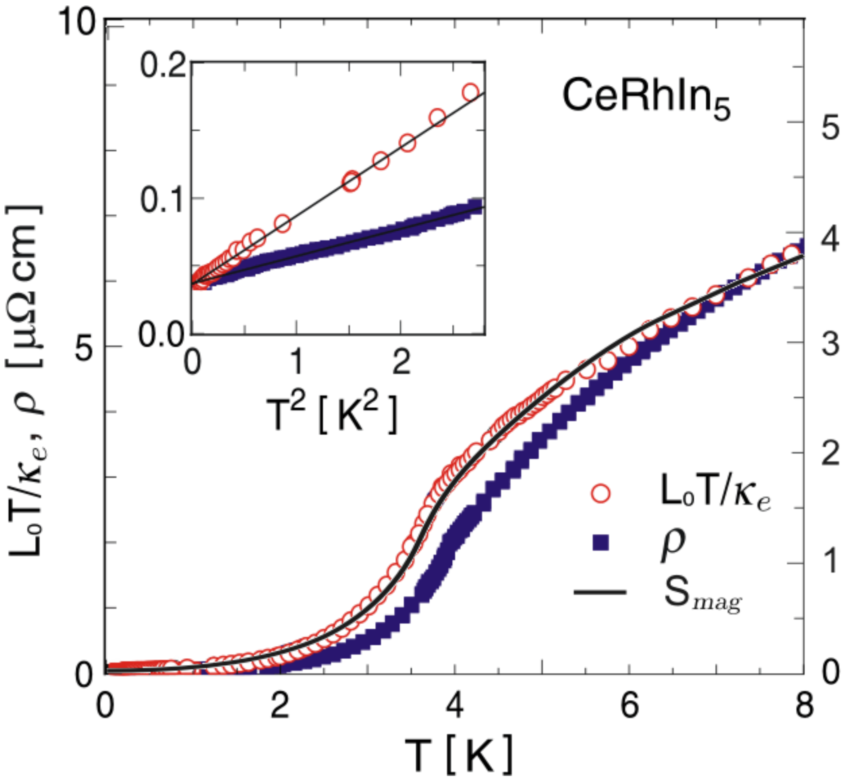
$$\kappa = \kappa_{\mathrm{electrons}} + \kappa_{\mathrm{phonons}}$$

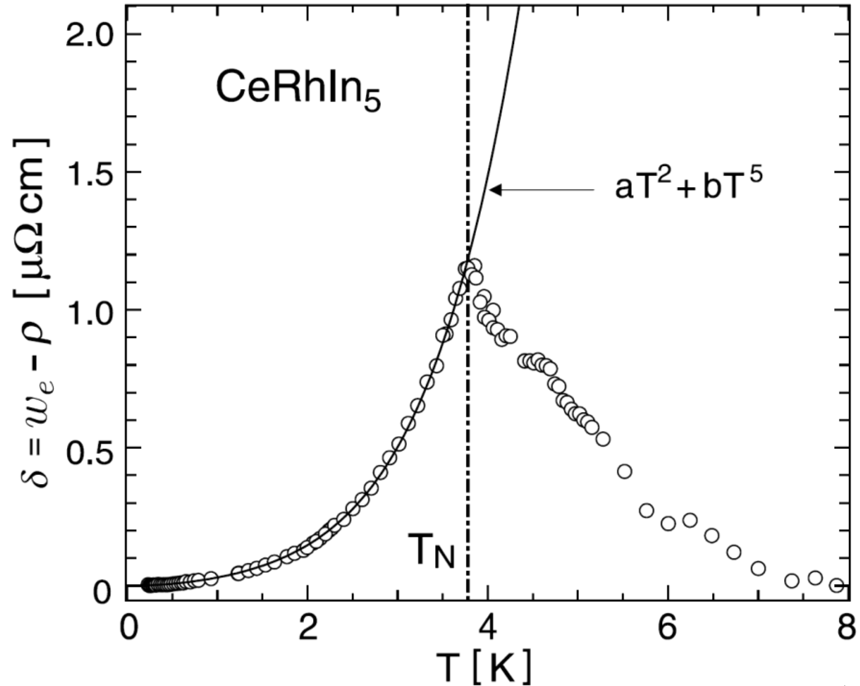




$CeRhIn_5$ heat & charge transport

Scattering mechanism





scattering ~ spin disorder

J. Paglione *et al.*, PRL **94**, 216602 (2005)

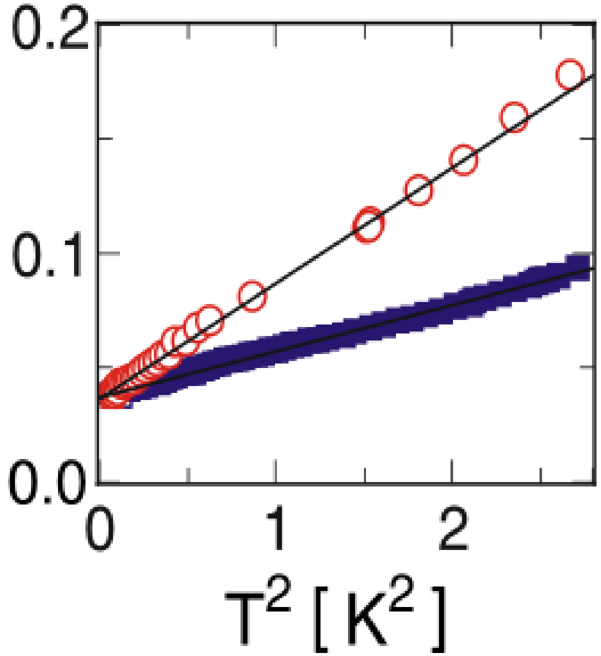
characteristic temperature T_{sf}

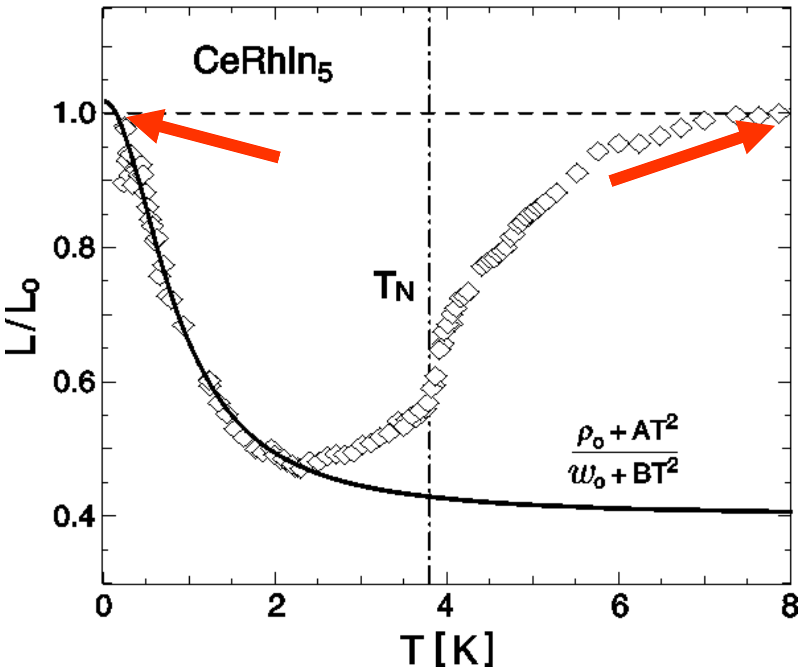
$CeRhIn_5$ heat & charge transport

Lorenz ratio $\kappa / \sigma T$

$T \rightarrow 0$: $L = L_0$

Wiedemann-Franz law



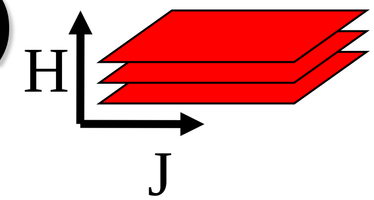


A new probe

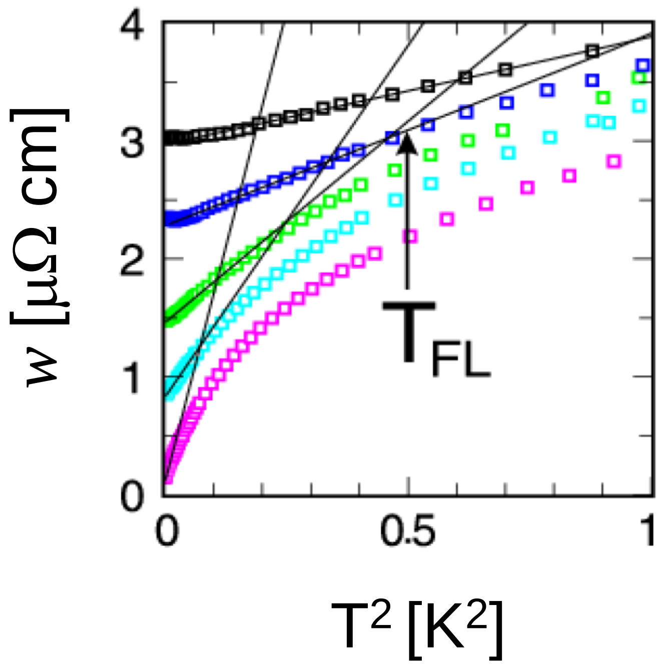
- $T \rightarrow 0$: test of WF law
- High T : T_{SF}

J. Paglione *et al.*, PRL **94**, 216602 (2005)

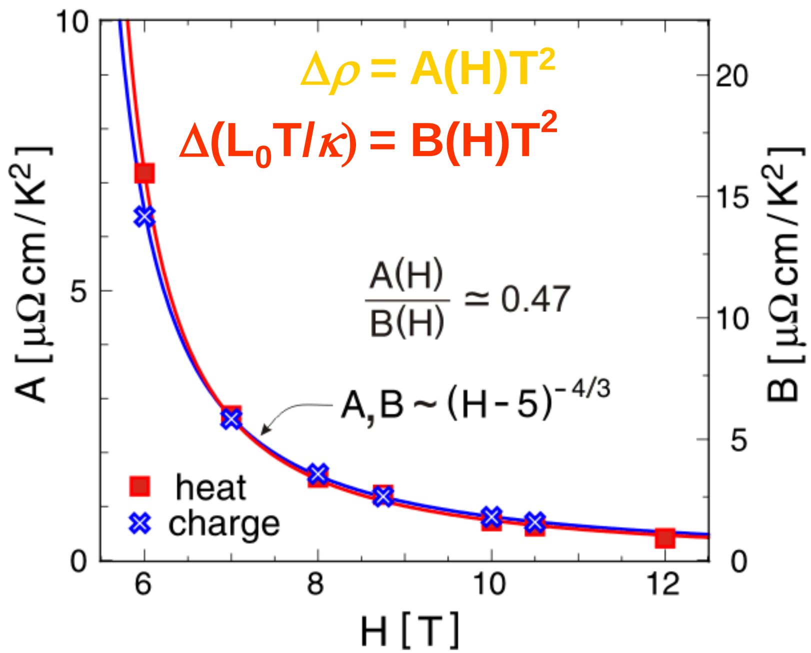
$CeCoIn_5$ In-plane transport (J//a)



Thermal resistivity



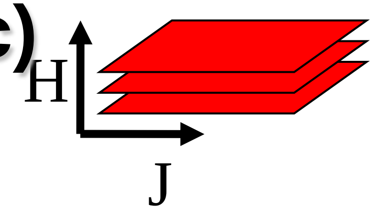
FL temperature T_{FL}



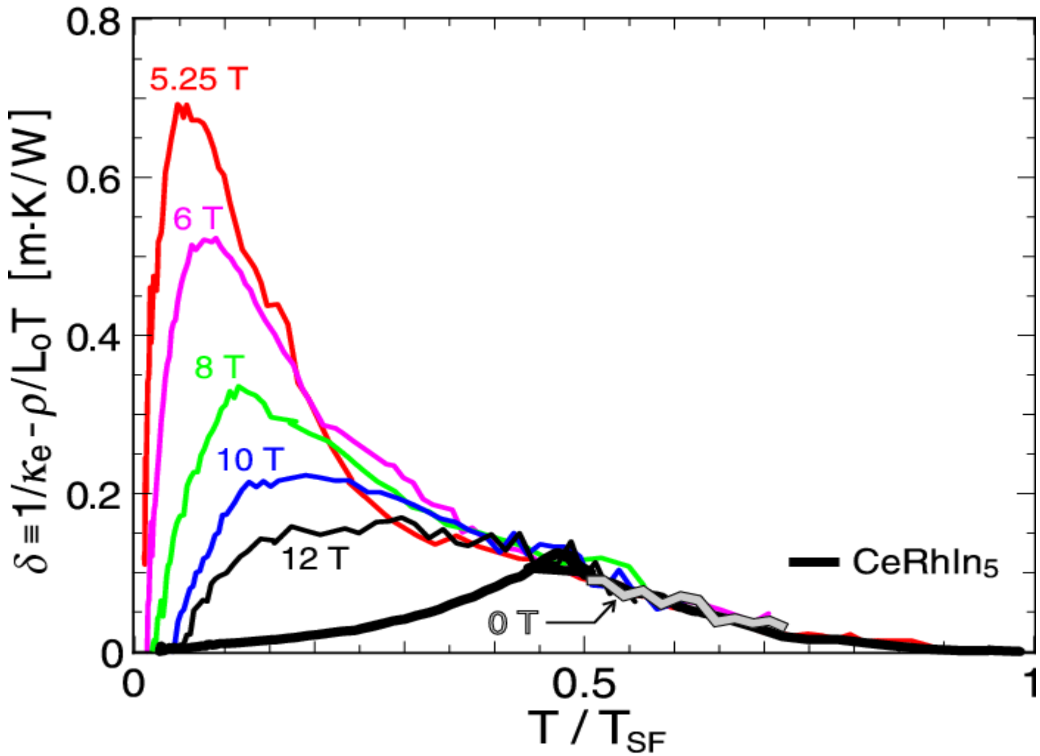
same critical exponent in
heat and charge transport

J. Paglione *et al.*, PRL97, 106606 (2006)

CeCoIn$_5$ In-plane transport ($J \perp c$)

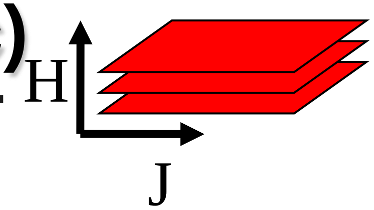


Difference in resistivities : thermal – electrical

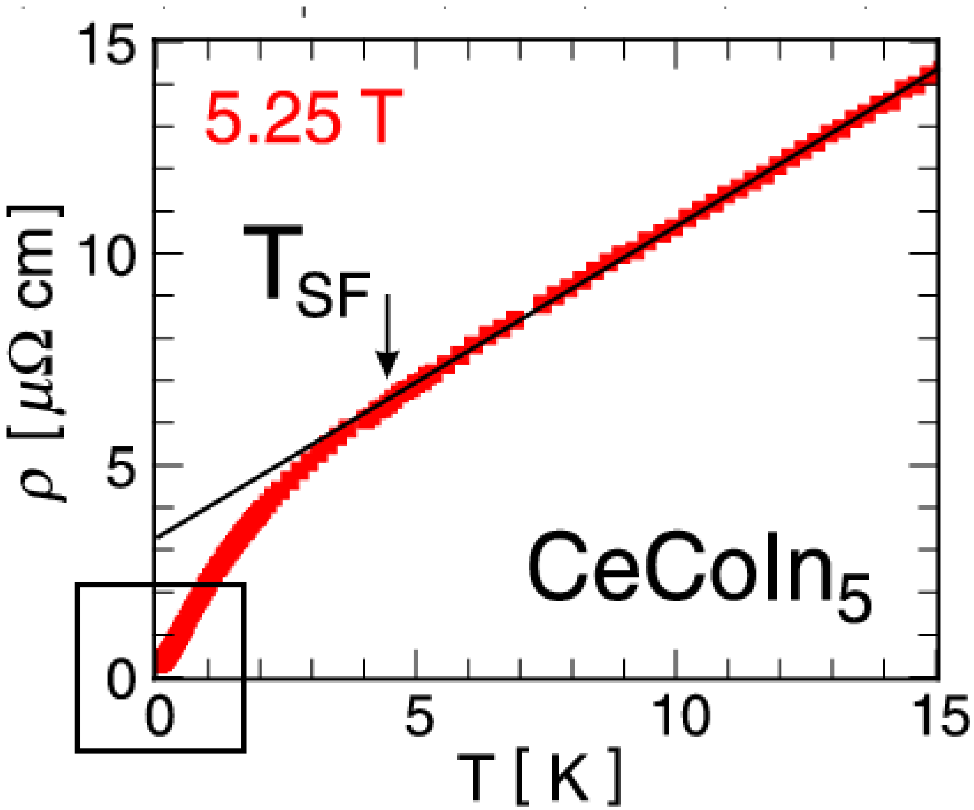


characteristic temperature T_{SF}

$CeCoIn_5$ In-plane transport ($J \perp c$)



H = H_c : electrical resistivity



What can be the cause of the violation?

Thermopower contribution to κ

$$\tilde{\kappa} = \kappa - T\sigma S^2$$

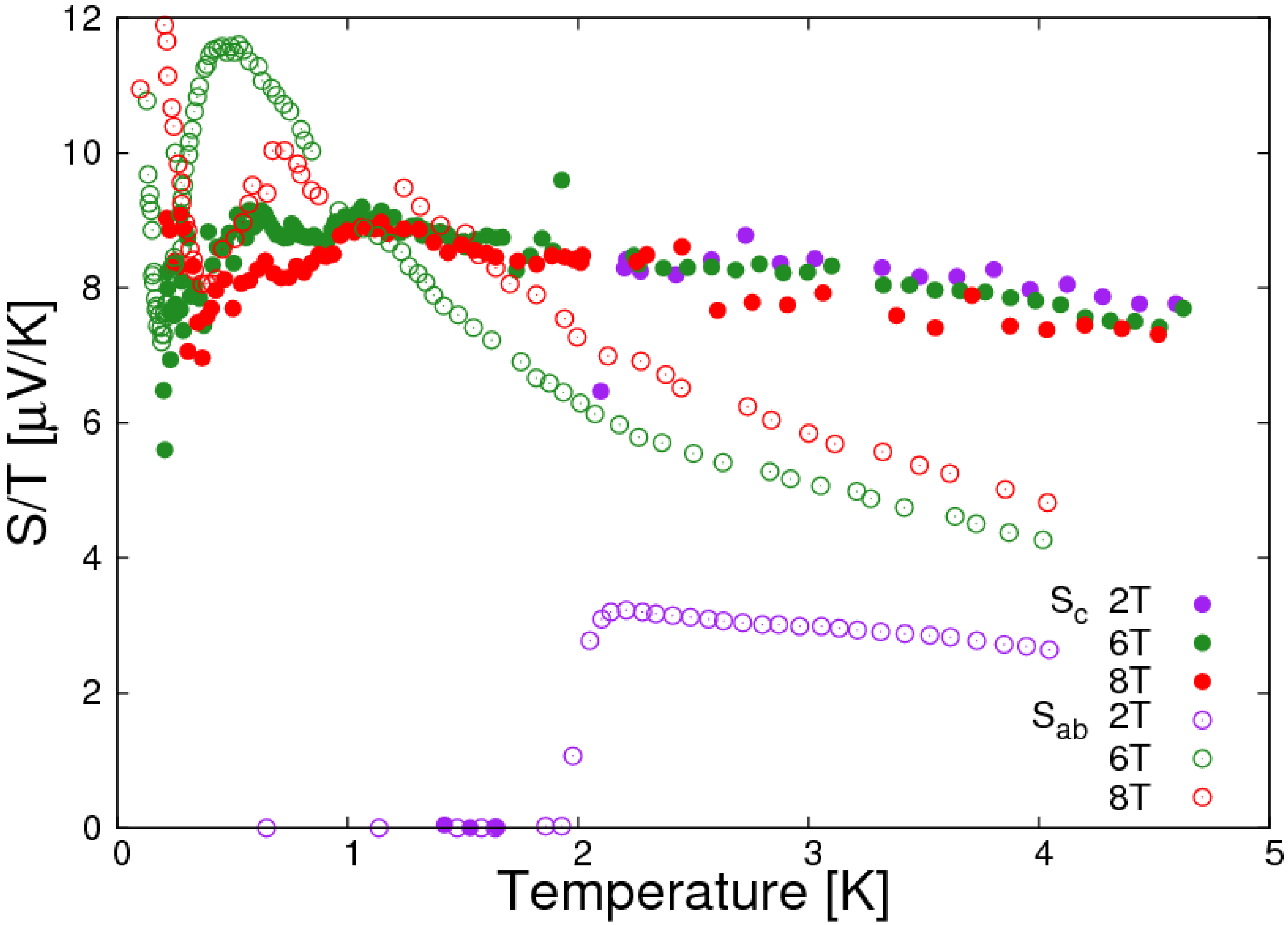
a. S/T ~ ln(1/T) Paul & Kotliar *PRB* **64**, 184414 (2001)

b. S/T ~ γ (FL, FM QCP)
   S/T << γ (AFM QCP)
   Miyake & Kohno *JPSJ* **74** 254 (2005)

c. S $\not\Rightarrow$ 0 Podolsky *et al* *PRB* **75** 014520 (2007)

d. S $\not\Rightarrow$ 0 Khodel *et al* *unpub.*

In metals S→0 when T →0 correction unimportant
What if not?



For heat current along c, S→0 when T →0 with constant slope
Very unusual for QCP!

Reading:
General
J. M. Ziman Principles of the theory of solids

Experimental details
F. Pobell Matter and Methods at Low temperatures
A. C. Anderson, Instrumentation at temperatures below 1K,
B. RSI 51, 1603 (1980)

Review articles on physics of thermal and thermoelectric phenomena
N. Hussey Adv. Phys. 51, 1685 (2002).
K. Behnia, D. Jaccard, J. Flouquet, JPCM 16, 5187 (2003)

Summary of transport measurements

Resistivity:
Good: direct information about conduction electrons
Bad: no quantitative theoretical description
Important info: charge gap (carrier density) and entropy (disorder, scattering)

Seebeck effect:
Good: charge carrier sign, density of states
Bad: no quantitative theoretical description, phonon drag
Important info: entropy per charge carrier

Hall effect:
Good: carrier charge, density and mobility (in combination with resistivity)
Good: analysis of multi-carrier transport
Bad: magnetic scattering
Important info:
carrier density

Magnetoresistance
Good: can distinguish multiple carrier case from single carrier case
Good: good theoretical description (Kohler rule)
Important info:
carrier density in multiple carrier conductivity
Magnetic scattering

Nernst effect:
Good: understanding multi-carrier situation
Bad: difficult to measure
Important info:
Multiband conductivity
exotic states of matter

Thermal conductivity:
Good: well understood theoretically
Bad: phonon contribution
Important info:
Charge scattering mechanism
characterization of unusual states of matter